Sanctus, Sanctus, Sanctus
Holy, holy, holy. The Lord Almighty is holy. His glory fills the world. Isaiah 6:3
We all strive to live holy lives at All Saints through
"… love that is patient and kind" that "never gives up"
enabling us to "Go out into the world, and love the people we meet."
(Saint Paul and Saint Teresa)

Christian Values to be found in our curriculum

Reverence	Wisdom	Thankfulness	Humility
Courage	Service	Compassion	Trust
Peace	Forgiveness	Welcome	Justice
Hope	Stewardship	Christian Fellowship	Love

All Saints Academy Policy for:

Calculation

Date of Policy: May 2020

Date of Review: May 2023

Responsibility of: The Governing Body and staff of All Saints Inter Church Academy

The policy was approved by the Governing Body on:

Written by: Mr L Crickwood

Contents

Appendices

1.0 Document History

Amendments and comments	Name of member of staff reviewing	Date
Version 1	Miss C Harrison	September 2012
Version 2 Complete rewrite of the policy.	Mrs J Holdershaw	January 2017
Version 3 Complete rewrite of the policy.	Mr L Crickwood	May 2020

2.0 Principles

At All Saints Inter-Church Academy we recognise that calculations are encountered by all children irrespective of age or stage of development. There is an acknowledged hierarchy in the strategies and skills needed for differing types of calculation but it is not intended to restrict the method or concept to specific age/year groups. It is however fundamental that children should have a core understanding in number concepts before beginning work on formal calculations.

Understanding of mathematical ideas happens when children make connections between real objects (manipulatives), symbols, language and pictures. This develops cognitive connections which enable children to make sense of new experiences by linking them to previous experiences. Wherever possible, practical apparatus should be available for children to use. As they become more confident and flexible in their thinking and they begin to apply their knowledge and understanding children should be taught how to select the most appropriate materials for a task. There should be a clear progression through concrete, pictorial and abstract activities.

3.0 Commitment to Equality

At All Saints Inter-Church Academy we believe that the provision of an outstanding education which develops a wide range of skills is crucial for opening up opportunities and increasing the chance of a successful life for every pupil. As an academy, we are therefore committed to avoiding discrimination and promoting equality at all levels and recognise that by doing this, attainment and progression for all pupils will be improved.

4.0 Legal requirements

The 2013 National Curriculum gave clear indications about the purpose and aims of maths within the curriculum. Great emphasis is given to the need for children to become fluent within the fundamentals of mathematics. This fluency will be developed through purposeful practise and practise with variation; enabling skills and concepts to be used and applied across contexts within and outside the sphere of maths.

5.0 Aims

this calculation policy aims to have in place a clear progression of calculation methods for All Saints Academy which is understood by all staff, visitors and parents and conforms to the National Curriculum guidelines.

6.0 Objectives

The objectives of this calculation policy are that all pupils:

- understand important concepts and make connections within mathematics.
- show high levels of fluency in performing written and mental calculations.
- are taught consistent calculation strategies.
- are ready for the next stage of learning.
- have a smooth transition between phases.
- are able to add, subtract, multiply and divide efficiently.
- are competent in fluency, reasoning and problem solving.

7.0 The Contribution to aspects of the Curriculum

Calculation is primarily linked with the Mathematics curriculum. However, at times, it will support other aspects of the curriculum, such as Science, Geography, History, PE, DT and Computing.

8.0 Mastery Planning Prompts

Develop Children's understanding of the = symbol

- Symbol = assertion of equivalence.
- Vary the position of the = symbol.
- Empty box problems.

Use empty box problems

- Used to promote reasoning and find easy ways to calculate.
- Use a sequence to develop conceptual connections.

Expect children to use the correct terminology and express reasoning

- Quality of reasoning is enhanced if children are consistently expected to use the **correct terminology** (eg. digit not number) and to explain their thinking in **complete sentences**.
- I say, you say, you say, you say, we all say (whole class chant the sentence).

Use intelligent practice (procedural variation)

- Children are required to reason and make connections between calculations. The connections made improve their fluency. Change one thing each time.

Identify difficult points

- Be aware of common misconceptions.
- Actively seek to uncover these.
- Visualizers are great to respond to misconceptions straight away.

Expose mathematical structure and work systematically

- Use images to see patterns.
- Make connections between models.
- Ask 'What is the same? What is different?'
- Show that the same structure can be applied to more complex numbers.

Move between the concrete and the abstract (conceptual variation)

- Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a lesson.

Contextualise the mathematics

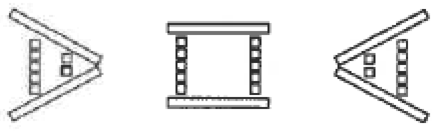
- Start with a context story.
- Keep returning back to the story.
- What does each number mean in context?
- Before, then, now.

Use questioning to develop mathematical reasoning

- Get children to explain how they worked out a calculation; value ideas but steer towards efficiency.
- What's the same, what's different?
- What do you notice?

Teach inequality alongside equality

- One way to introduce this is to use rods and cubes to make concrete representation (balance scales can also be used).



< and > can also help deepen understanding of key concepts,
eg 18p £0.15

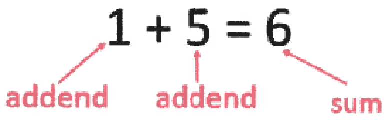
9.0 Year Group Overview

Year Group	Addition	Subtraction	Multiplication	Division
Year 1	Number bonds to 20 Number bonds within 20 =n+n n+n= Introduce inequality (> or <)	Number bonds within 20 20-1 digit= 20-2 digits (less than 20) = =n-n n-n=	Multiplication through repeated addition	Grouping and sharing objects and pictures.
Year 2	1 digit + 1 digit + 1 digit 2 digits + 1 digit 2 digits + 2 digits (∑<100) Addition of fractions with the same denominator within a whole. Part, Part, Whole models Bar models	2 digits – 1 digit 2 digits – Multiples of 10 2 digits – 2 digits (with borrowing later in the year) Subtraction of fractions with the same denominator within a whole. Part, Part, Whole models Bar models	2, 5 and 10 multiplication facts Repeated addition Arrays	2, 4, 5 and 10 division facts Grouping and sharing Repeated subtraction 2 digit ÷ 1 digit Word problems including division
Year 3 Introduction to subject specific vocabulary	2 digits + 2 digits (∑>100) 3 digits + 3 digits Column addition Addition of fractions with the same denominator. Part, Part, Whole models Bar models	2 digits – 2 digits with borrowing 2 digits – Multiples of 10 3 digits – 2 digits Column subtraction Subtraction of fractions with the same denominator. Part, Part, Whole models Bar models	2, 5, 10, 3, 4, 6, and 8 multiplication facts 2 digits x 1 digit Arrays Column method	2, 5, 10, 3, 4, 6, and 8 division facts 2 digits ÷ 1 digit Grouping and sharing Remainders (r)
Year 4	Up to 4 digits + 4 digits Column method Addition of fractions with different denominators.	Up to 4 digits – 4 digits. Introduction to constant difference. Column method. Subtraction of fractions with different denominators.	Multiplication facts up to 12 x 12 2 digits x 1 digit 2 digits x 2 digits Column method Multiplying a fraction by another fraction.	Division facts up to 144 ÷ 12 3 digits ÷ 1 digit 'Bus stop' method Remainders (r) Dividing a fraction by another fraction.
Year 5	Up to 5 digits + 5 digits Addition of decimals Column method Addition of fractions with different denominators.	Up to 5 digits - 5 digits Constant difference Subtraction of decimals Column method Subtraction of fractions with different denominators.	1 digit x 1 digit x 1 digit 3 digits x 2 digits 4 digits x 2 digits Column method Multiplying a fraction by another fraction. Multiplying a fraction by a whole number.	3 digits ÷ 1 digit 4 digits ÷ 1 digit Remainders as a fraction or a decimal Dividing a fraction by another fraction. Dividing a fraction by a whole number.
Year 6	Greater than 5 digits + 5 digits Addition of decimals Column method Multi-step word problems Addition of a fraction with a mixed number. Addition of two mixed numbers.	Greater than 5 digits - 5 digits Constant difference Subtraction of decimals Column method Multi-step word problems Subtraction of a mixed number with a fractions. Subtraction of two mixed numbers.	1 digit x 1 digit x 1 digit 3 digits x 2 digits 4 digits x 2 digits Multiplying decimals Column method Multiplying a fraction by another fraction. Multiplying a fraction by a whole number. Multiplying a fraction by a mixed number. Multiplying two mixed numbers together.	4 digits ÷ 1 digit 4 digits ÷ 2 digits Remainders as a fraction or a decimal Dividing a fraction by another fraction. Dividing a fraction by a whole number. Dividing a mixed number by a fraction.

10.0 Addition

Key vocabulary: sum, total, parts, whole, plus, add, altogether, more, is equal to, is the same as

Subject specific vocabulary: Augend + Addend = Sum Addend +Addend = Sum



Contextualise the mathematics
- WHAT DOES THIS NUMBER REPRESENT?

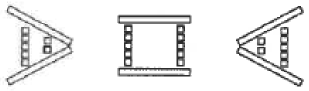
Expose mathematical structure and work systematically

Expect children to use correct terminology and express reasoning
- Use **STEM SENTENCES**
- Answer in **complete sentences**

Identify difficult points
- Be aware of common misconceptions
- Actively seek to uncover these

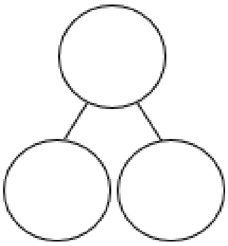
Move between the concrete, pictorial and the abstract (CPA)

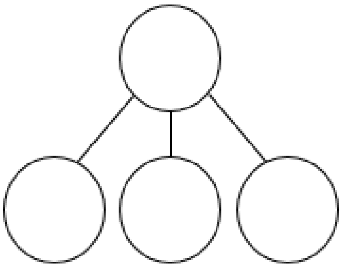
Teach inequality alongside equality

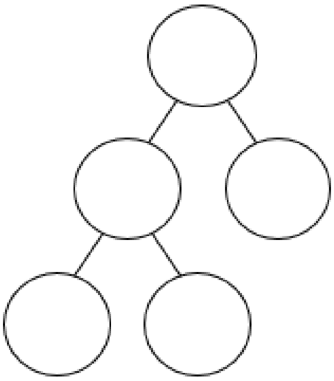


- < and > can also help deepen understanding of key concepts, eg 18p £0.15

Use empty box problems
- Promotes reasoning and finding easy ways to calculate
- Use a sequence to develop conceptual connections







	5	6	2	1	
+	5	7		7	9
1		3	2		8

	1	6	2		
+	8	8	7	7	9
1				9	5

10.1 The Big Ideas of Addition

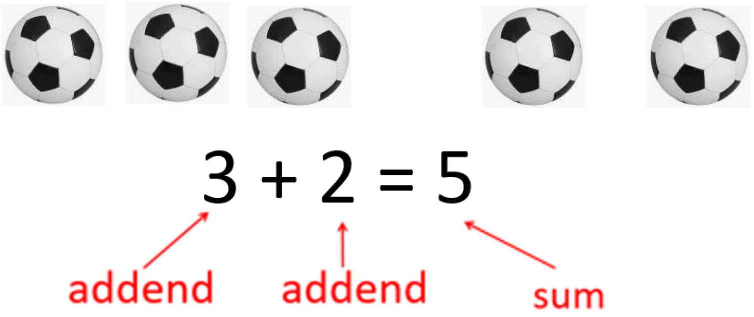
addend + addend = sum

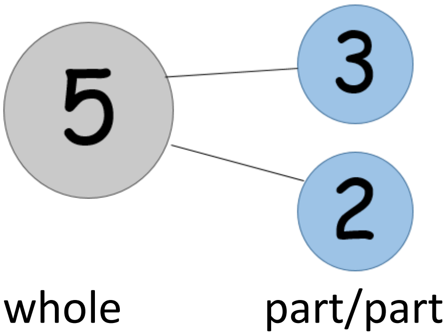
There are two structures of addition: aggregation and augmentation.

Aggregation structure

Combining two or more parts to make a whole is called **aggregation.**

Ben had 3 footballs and Zoe had 2 footballs. How many footballs are there altogether?





The sum of the parts is equal to the whole.

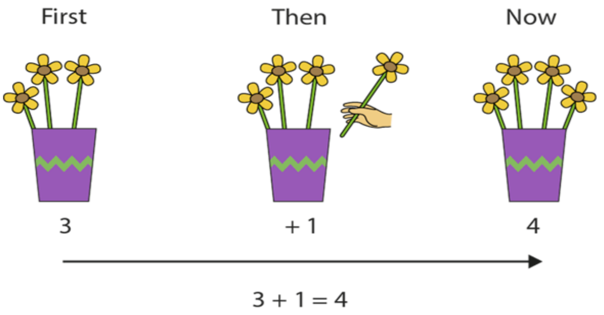
Addition is commutative because the parts can be added in any order.

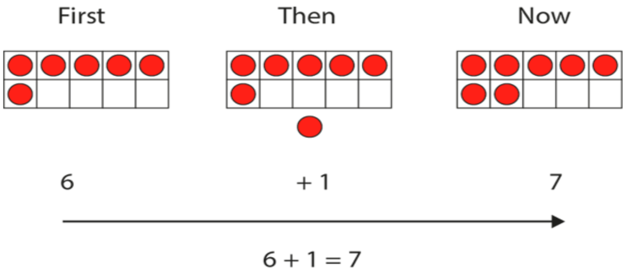
Augmentation structure:

An addition context described by a **first, then, now** story is an example of **augmentation**.

Harry had 3 footballs, then he was given 2 more. How many does he have now?

Both structures can be represented on a part/whole diagram.





When formal written methods are introduced, please encourage children to continue to use **NUMBER SENSE**.

Stop, think, consider the numbers involved in the calculation before choosing an efficient method for solving.

245 + 98 could be solved by adjusting + 100 and subtracting 2 rather than using a column method.

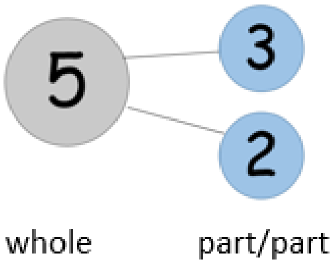
Prior to calculating, start with a **stem sentence** "I think that the best way of working this out …"

Simple numbers are used to teach formal algorithms initially. 23 + 14 can be worked out mentally but is used to show how the algorithm works. We are not suggesting that a column method is usually used for this calculation.

At All Saints Academy, **carried figures are put at the bottom of the columns** (NB. White Rose puts carried figures at the bottom.)

10.2 Addition Stem sentences

A sample of Stem sentences which could be used with addition. This list is not complete.



A whole can be broken into a number of parts.

The sum of the parts is equal to the whole.

We can add the parts in any order. *(Addition is associative)*

We can only add things with the same noun. *(proto-algebra)*

If you change the order of the addends, the sum remains the same.
(Addition is commutative)

In addition, we can add to one set to make it bigger. The total is the sum. *(Augmentation structure*)

In addition, we can combine one or more sets. The total is the sum. *(Aggregation structure)*

One part is _____, the other part is _____. The Whole is ______.

The augend is _______, and the addend is ________ . The sum will be ________.

The first addend is _______, the second addend is _______ . The sum will be_______.

Teacher notes are in italics.

10.3 Year Group Breakdown

Reception Objectives

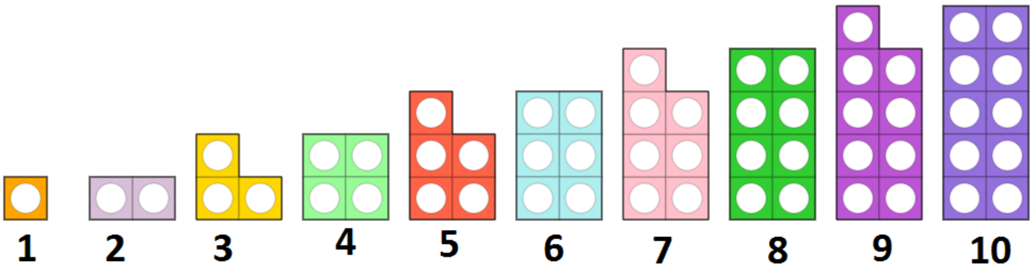
- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Key Skills:
Adding 0 and 1 to a number Addition bonds within 10 e.g. 5=4+1 Addition bonds to 10

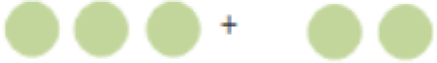
If available, Numicon shapes are introduced straight away and can be used to:

- identify 1 more/less
- combine pieces to add.
- find number bonds.
- add without counting.

Children can record this by printing or drawing around Numicon pieces.



Children begin to combine groups of objects using concrete apparatus



Once children are confident in using concrete apparatus they then move onto drawing their own circles or pictures to help support the combining of amounts and are taught how to count on from the biggest number.

Construct number sentences verbally or using cards to go with practical activities.

Children are encouraged to read number sentences aloud in different ways
"Three add two equals 5" "5 is equal to three and two"

Children make a record in pictures, words or symbols of addition activities already carried out.

Solve simple problems using fingers

Number tracks can be introduced to count up on and to find one more:

1	2	3	4	5	6

What is 1 more than 4? 1 more than 13?

Number lines can then be used alongside number tracks and practical apparatus to solve addition calculations and word problems.

Children will need opportunities to look at and talk about different models and images as they move between representations.

Year 1 Objectives

- Number bonds to, and related facts within, 20.
- Add one and two digit numbers to 20, including zero.

Key Skills:
Adding 0 and 1 to a number Addition bonds within 10 e.g. 5=4+1 Addition bonds to 10

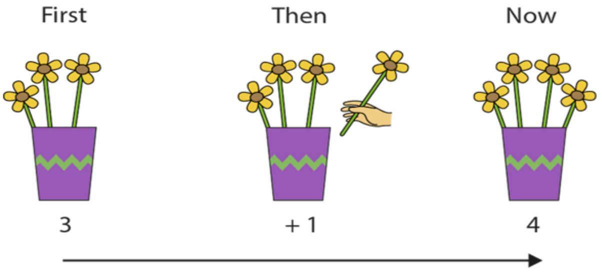
Start with expressions (no = sign)

2 + 4

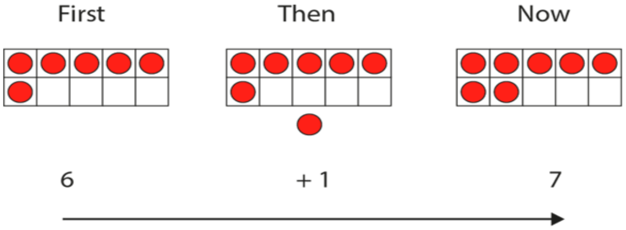
4 + 2

Move on to equations (has = sign)

5 = 3 + 2

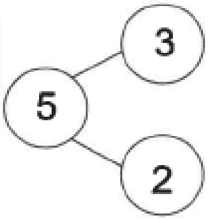


3 + 1 = 4

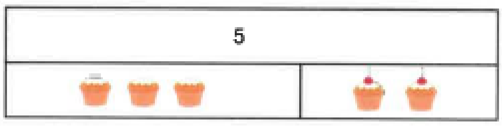


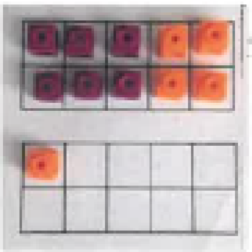
6 + 1 = 7

Use part whole diagram (include zero)
Zero is not a part

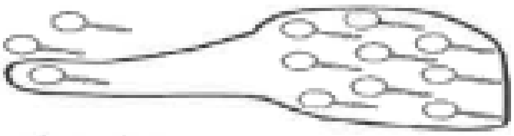


Teacher to use the bar model in summer term

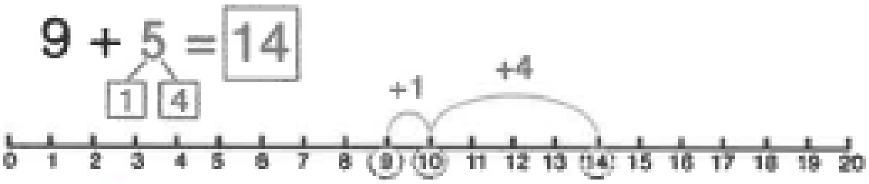




Start with the bigger number and use the smaller number to make 10.



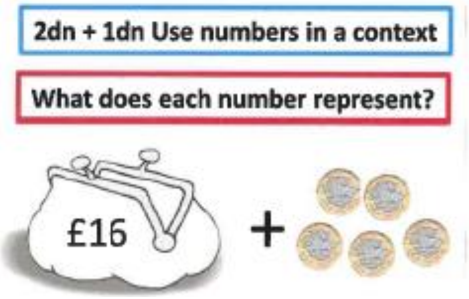
3 + 9 =

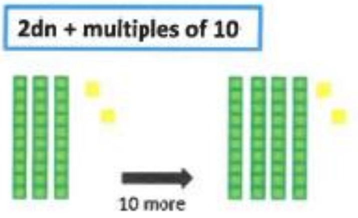


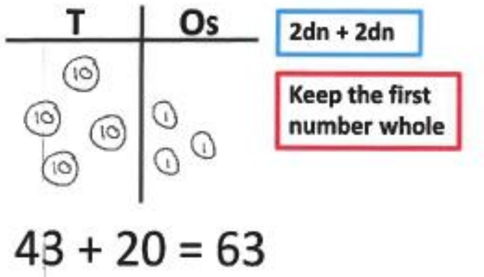
Year 2 Objectives:

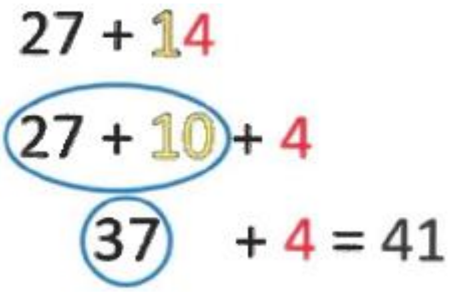
- Add three one-digit numbers together.
- Add a two digit number to a one digit number
- Add two two-digit numbers together where the sum is less than one hundred.
- Add two fractions with the same denominator within a whole.

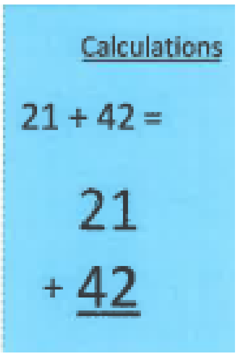
Key Skills:
Adding a two digit number to a one digit number. Add a two digit number to a multiple of ten.











Calculations

21 + 42 =

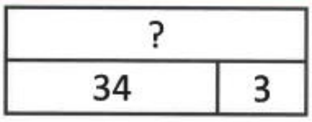
$$\begin{array}{r} 21 \\ +\underline{42} \end{array}$$

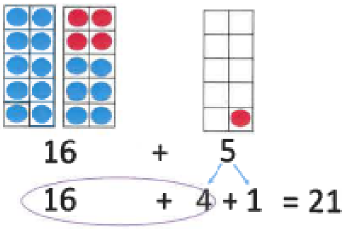
2dn + 1dn Use numbers in a context

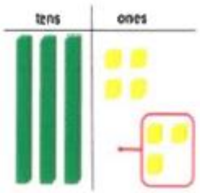
At first Fiona had saved £34 and then she added her £3 pocket money to that.

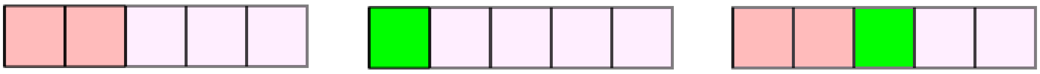
How much does she have now?

Children to use the bar model









$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

Year 3 Objectives

- Add two two-digit numbers together where the sum is greater than one hundred.
- Add two three-digit numbers together
- Add two fractions with the same denominator.

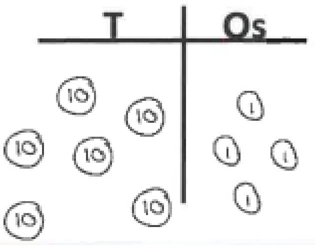
Key Skills:
Adding a two digit number to a one digit number. Add a two digit number to a multiple of ten. The column method

2dn + 1dn

26 + 5

26 + 4 + 1 = 31

2dn + multiples of 10



64 + 30 = 94

2dn + 2dn

Keep the first number whole

27 + 14

27 + 10 + 4

37 + 4 = 41

2dn + 2dn with renaming Carried figure at the top

$$\begin{array}{r} {}^{+1} \\ 48 \\ +\ 14 \\ \hline 62 \\ \hline \end{array}$$

3dn + 3dn with renaming Carried figure at the top

$$\begin{array}{r} {}^{+1}\,{}^{+1} \\ 258 \\ +\ 165 \\ \hline 423 \\ \hline \end{array}$$

Solve missing box problems

$$\begin{array}{r} 58 \\ +\ 1\boxed{} \\ \hline 75 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ +\ 14 \\ \hline 37 \\ \hline \end{array}$$

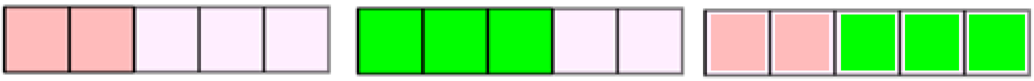
Column method

Unitise:

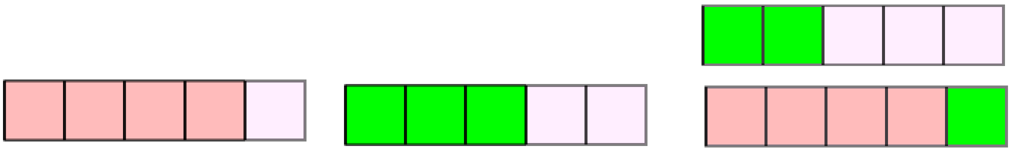
8 ones + 4 ones equals 12 ones. We rename this: it is 1 ten and 2 ones.

4 tens add 1 ten add the 1 carried ten equals 6 tens (not 40 + 10 + 10 = 60)

Children to use the bar model



$$\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$$



$$\frac{4}{5} + \frac{3}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Year 4 Objectives

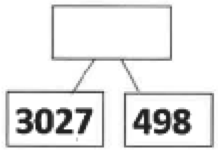
- Add numbers up to four digits.
- Add two fractions with the different denominator.

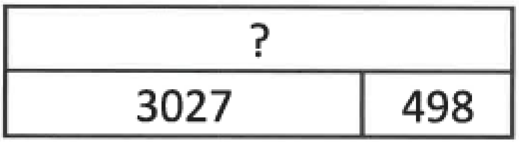
Key Skills:
Adding a two digit number to a one digit number. Add a two digit number to a multiple of ten. The column method

Column method

Unitise:
8 ones + 4 ones equals 12 ones. We rename this: it is 1 ten and 2 ones.
4 tens add 1 ten add the 1 carried ten equals 6 tens (not 40 + 10 + 10 = 60)

Children to use the part whole and bar model to develop estimation and number sense





3dn + 3dn with renaming Carried figure at the top

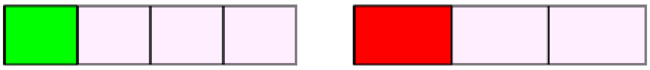
$$\begin{array}{r} {}^{+1\;+1} \\ 258 \\ +\,165 \\ \hline 423 \\ \hline \end{array}$$

4dn + 4dn with renaming Carried figure at the top

$$\begin{array}{r} {}^{+1\;+1\;+1} \\ 7289 \\ +\,5145 \\ \hline 12434 \\ \hline \end{array}$$

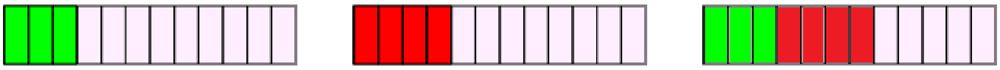
Solve missing box problems

$$\begin{array}{r} 7\,5\,8 \\ +\,\blacksquare\,1\,5 \\ \hline 1\,0\,\blacksquare\,3 \\ \hline \end{array}$$



$$\frac{1}{4} + \frac{1}{3}$$

Convert the fractions so that the denominators are the same



$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

Year 5 Objectives:

- Add numbers up to five digits.
- Addition of two decimal numbers.
- Add two fractions with the different denominator.

Column method

Unitise:

8 ones + 4 ones equals 12 ones. We rename this: it is 1 ten and 2 ones.
4 tens add 1 ten add the 1 carried ten equals 6 tens (not 40 + 10 + 10 = 60)

Decimal numbers
Different number of digits

$$\begin{array}{r} {}^{+1} \\ 57.30 \\ +\ \ 6.08 \\ \hline 63.38 \\ \hline \end{array}$$

- **Vary the number of digits in the number**
- **= sign on the RHS**
- **Balanced equations**

65 + 577 =

? = 4277 + 656

648 + ? = 1036 + 58

Problem solving

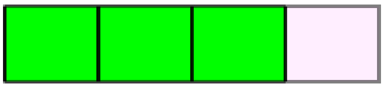
Amy and Matthew are playing their favourite
computer game. Amy's current high score is 8,524.
Matthew's high score is bigger than Amy's and when
you add them together their combined total is19,384.
What is Matthew's high score?

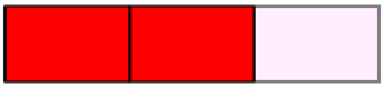
Work out the missing numbers.

$$\begin{array}{r} \square 4 \square 3 \square \\ +\ 2 \square 5 \square 2 \\ \hline 78529 \\ \hline \end{array}$$

Children to use the part whole and bar model to develop estimation and number sense

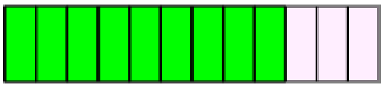
?	
375.5	14.3

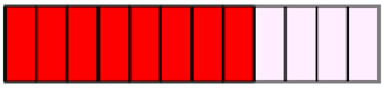




$$\frac{3}{4} + \frac{2}{3}$$

Convert the fractions so that the denominators are the same.





$$\frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

Year 6 Objectives

- Add numbers greater than five digits.
- Addition of two decimal numbers.
- Add a fraction with a mixed number.
- Add two mixed numbers.

- **Vary the number of digits in the number**
- **= sign on the RHS**
- **Balanced equations**

247 + 14,699 =

? = 6.9 + 14.32

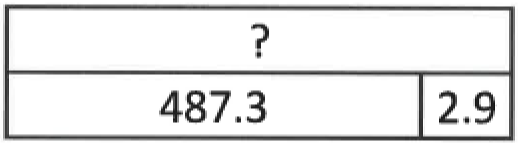
$\frac{2}{5} + \frac{3}{10} + \frac{1}{2} =$

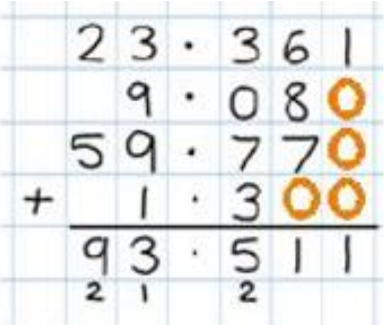
Problem solving

A is an odd number which rounds to 100,000 to the nearest ten thousand.
It has a digit total of 30.
B is an even number which rounds to 500,000 to the nearest hundred thousand.
It has a digit total of 10.
A and B are both multiples of 5 but end in different digits.

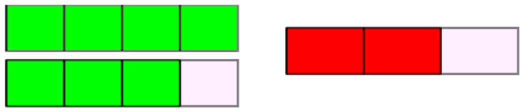
A	B
631,255	

Children to use the part whole and bar model to develop estimation and number sense



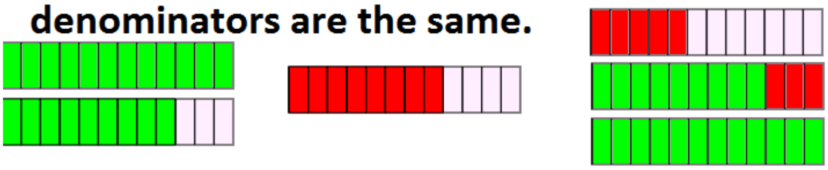


$$\begin{array}{rr} & 23.361 \\ & 9.080 \\ & 59.770 \\ + & 1.300 \\ \hline & 93.511 \\ & {}_{2\,1\;\;2} \end{array}$$

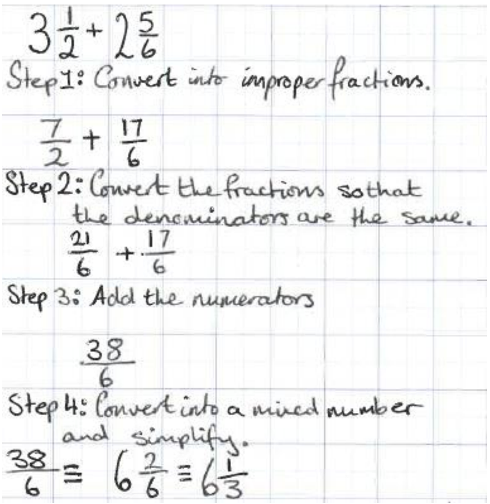


$1\frac{3}{4}$ $\left(\frac{7}{4}\right)$ + $\frac{2}{3}$

Convert the mixed number into an improper fraction. Convert the fractions so that the denominators are the same.



$\frac{21}{12} + \frac{8}{12} = \frac{29}{12} = 2\frac{5}{12}$



$3\frac{1}{2} + 2\frac{5}{6}$

Step 1: Convert into improper fractions.

$\frac{7}{2} + \frac{17}{6}$

Step 2: Convert the fractions so that the denominators are the same.

$\frac{21}{6} + \frac{17}{6}$

Step 3: Add the numerators

$\frac{38}{6}$

Step 4: Convert into a mixed number and simplify.

$\frac{38}{6} = 6\frac{2}{6} = 6\frac{1}{3}$

11.0 Subtraction

Key vocabulary: take away, less than, the difference, subtract, minus, fewer, decrease

Subject specific vocabulary: Minuend – Subtrahend = Difference

$$8 - 1 = 7$$

minuend subtrahend difference

Contextualise the mathematics

- WHAT DOES THIS NUMBER REPRESENT?

Expose mathematical structure and work systematically

Expect children to use correct terminology and express reasoning

- Use **STEM SENTENCES**
- Answer in **complete sentences**

Identify difficult points

- Be aware of common misconceptions
- Actively seek to uncover these

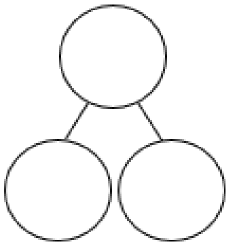
Move between the concrete, pictorial and the abstract (CPA)

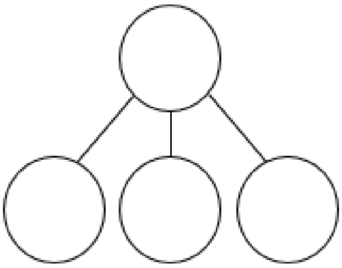
Teach inequality alongside equality

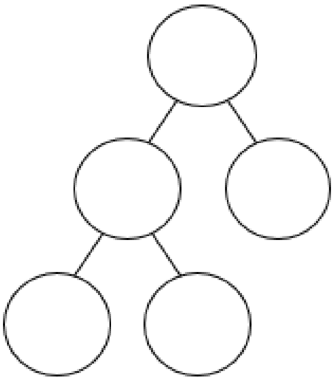
- < and > can also help deepen understanding of key concepts, eg 18p £0.15

Use empty box problems

- Promotes reasoning and finding easy ways to calculate
- Use a sequence to develop conceptual connections







	5	6	2	1	2
-		7		7	
	3		0		8

			4		7
-	4	2		3	5
	2	8	8	9	2

11.1 The Big Ideas of Subtraction

Minuend – Subtrahend = Difference

There are three structures of subtraction: partitioning, reduction and difference.

Partitioning structure

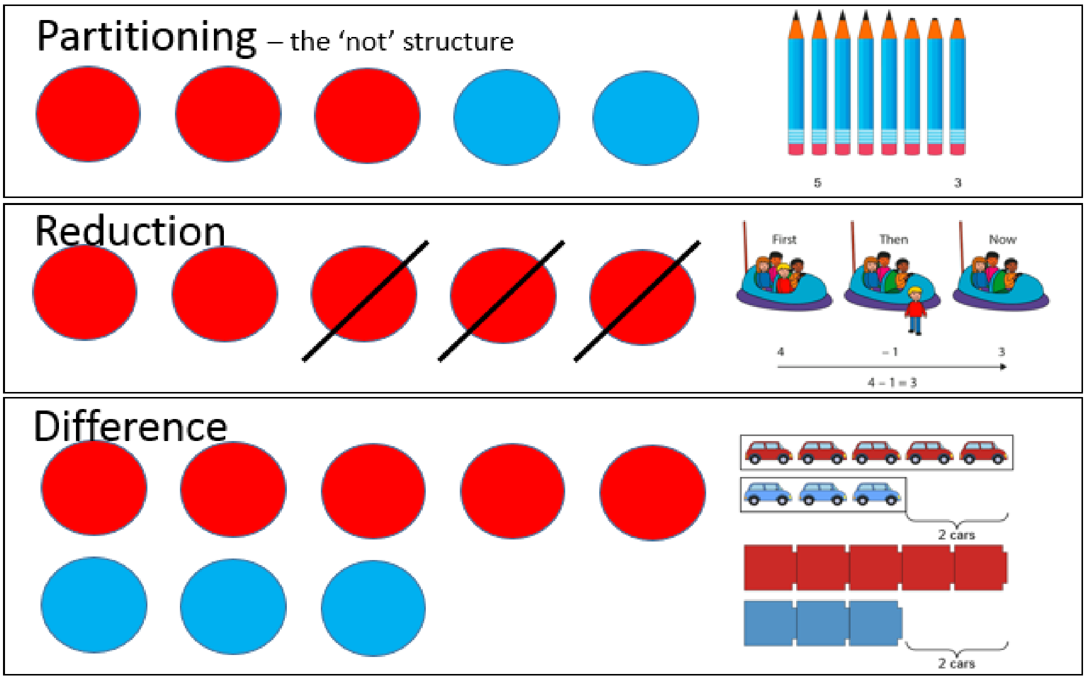
Sometimes called the ‘not’ structure. Splitting the whole into two or more parts is called **partitioning.**

Reduction structure

Removing a part from the whole to find the other part is called **reduction.**

Difference structure

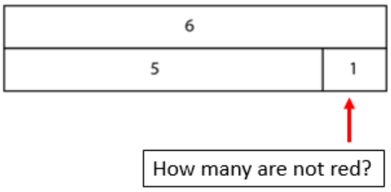
Comparing two or more wholes and finding how many more or less one of the wholes have is called finding the **difference.**



Partitioning:

There are 6 flowers in a vase, 5 are red. How many are not red?

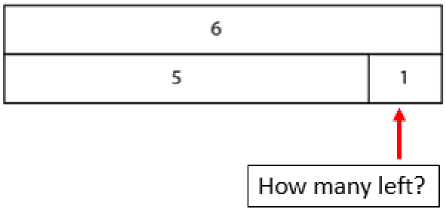
6 is the whole. 5 is a part. 1 is a part.



Reduction:

There are 6 flowers in a vase. I take out 5. How many have I got left?

6 is the whole. 5 is a part. 1 is a part.



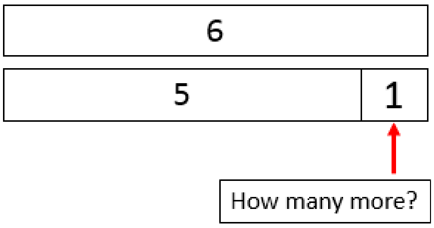
Difference:

Sarah has 6 flowers in a vase.   Gary has 5 flowers in a vase.

How many more flowers does Sarah have?

This is a comparative structure of subtraction.

It can be represented clearly on a bar model.



11.2 Subtraction Stem sentences

A sample of Stem sentences which could be used with subtraction. This list is not complete.

The whole can be split into parts.

The sum of the parts is equal to the whole.

Whole subtract a part equals a part.

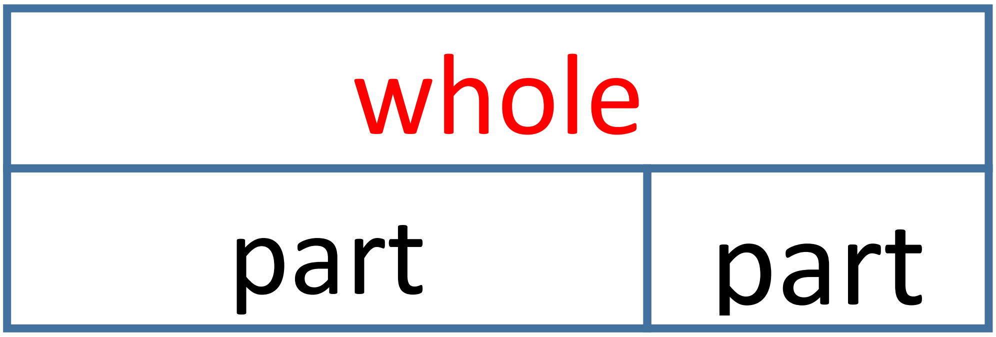
Subtraction cannot be done in any order as we cannot swap the whole and the part.

The minuend is the whole. The subtrahend is a part. The difference is a part.

The minuend is ________, the subtrahend is __________. The difference is _________.

The minuend is ________, the difference is __________. The Subtrahend is _________.

If you **change** the **minuend** and the **subtrahend** by the **same amount**, the **difference** will remain the same.



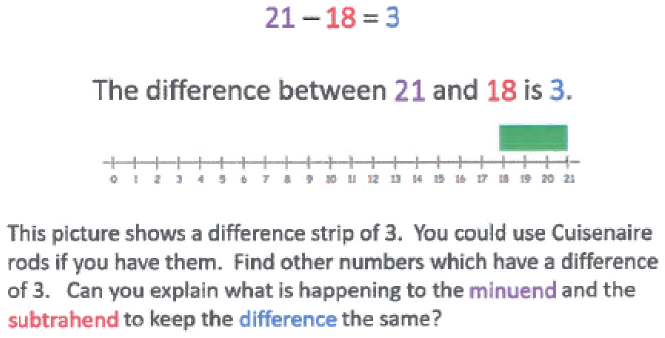
11.3 The Principle of Constant Difference

If you **change** the **minuend** and the **subtrahend** by the **same amount**, the **difference** will remain the same.

53-19 = 54-20

This subtraction principle is taught from Year 4 at All Saints Academy. Children need lots of practical experience to understand the principle and then be given chance to recognise calculations where this strategy is particularly effective.

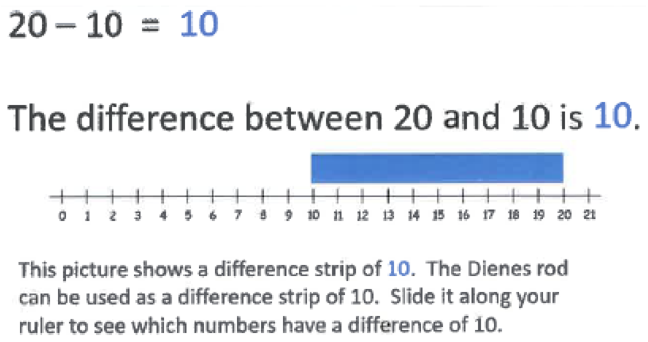
Teaching the Principle of Constant Difference



21 – 18 = 3

The difference between 21 and 18 is 3.

This picture shows a difference strip of 3. You could use Cuisenaire rods if you have them. Find other numbers which have a difference of 3. Can you explain what is happening to the minuend and the subtrahend to keep the difference the same?



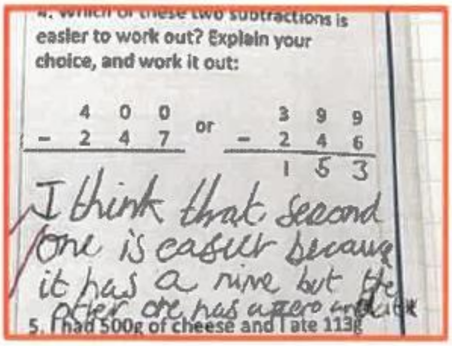
20 – 10 = 10

The difference between 20 and 10 is 10.

This picture shows a difference strip of 10. The Dienes rod can be used as a difference strip of 10. Slide it along your ruler to see which numbers have a difference of 10.

Applications of the constant difference

The power of the principle is that it extends naturally to subtraction that are procedurally more demanding and/or conceptually more challenging: in particular, subtractions with non-integer terms in KS2.

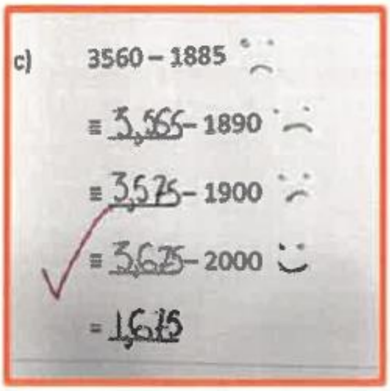


4. Which of these two subtractions is easier to work out? Explain your choice, and work it out:

400 − 247 or 399 − 246 = 153

I think that second one is easier because it has a nine but the other one has a zero and it

5. I had 500g of cheese and I ate 113g



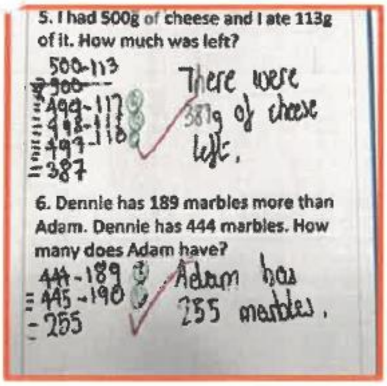
c) 3560 – 1885

≡ 3,565 – 1890

≡ 3575 – 1900

≡ 3625 – 2000

≡ 1625



5. I had 500g of cheese and I ate 113g of it. How much was left?

500-113

~~500~~

= 499-117

= 498-116

= 497-1~~1~~0

= 387

There were 387g of cheese left.

6. Dennie has 189 marbles more than Adam. Dennie has 444 marbles. How many does Adam have?

444 - 189

= 445 - 190

= 255

Adam has 255 marbles.

6 – 3.7 ≡ 6.3 - 4

11.4 Year Group Breakdown

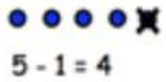
Reception Objectives

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.
- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Key Skills:
Adding 0 and 1 to a number Addition bonds within 10 e.g. 5=4+1 Addition bonds to 10

Children begin with mostly pictorial representations

Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.
Concrete apparatus models the subtraction of 2 objects from a set of 5.



Construct number sentences verbally or using cards to go with practical activities.

Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one"

Children make a record in pictures, words or symbols of subtraction activities already carried out.

Solve simple problems using fingers

Number tracks can be introduced to count back and to find one less:

1	2	3	4	5	6

What is 1 less than 9? 1 less than 20?

Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line.

Children will need opportunities to look at and talk about different models and images as they move between representations.

Year 1 Objectives

- Number bonds and related number facts within 20.
- Subtract one and two digit numbers within 20, including zero.

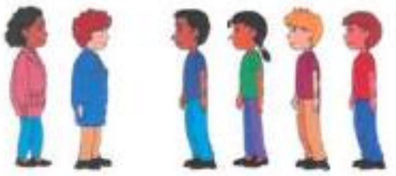
Key Skills:
Subtraction bonds within 10. Subtraction bonds from 10. Subtracting zero and one from a number.

Subtraction is not commutative

Start with expressions (no = sign)

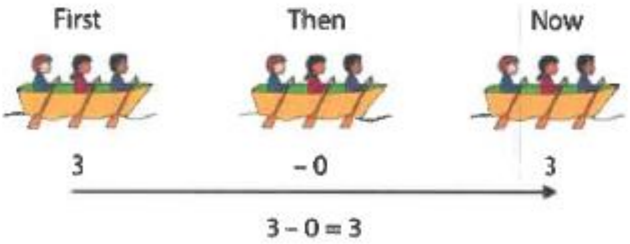
8 - 3

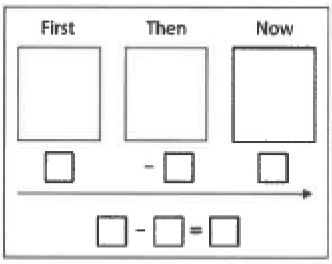
Move on to equations (has = sign)
Partitioning

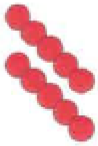


6 - 2 = 4

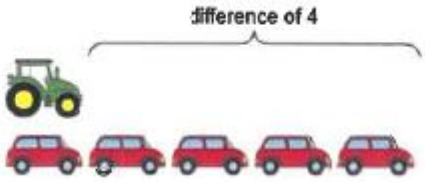
Reduction



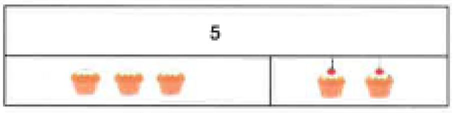




Difference

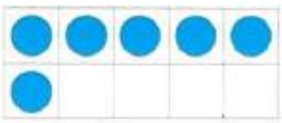


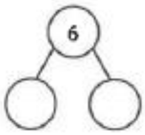
Teacher to use the bar model in the summer term

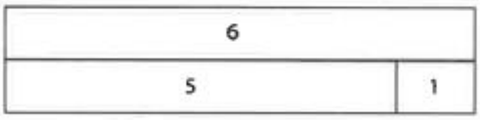


Use part whole diagram (include zero)

Partitioning single digit numbers







Year 2 Objectives:

- Subtracting a one digit number from a two digit number.
- Subtracting multiples of ten from a two digit number.
- Subtracting a two digit number from another two digit number.
- Subtracting fractions with the same denominator within a whole.

Key Skills:
Subtracting a one digit number from a two digit number. Subtracting multiples of ten from a two digit number.

Subtraction is not commutative

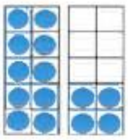
2dn - 1dn Use numbers in a context

What does each number represent?

2dn - 1dn Use numbers in a context

At **first** Fiona had £24 and **then** she spent £5.

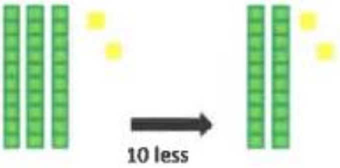
How much does she have **now**?

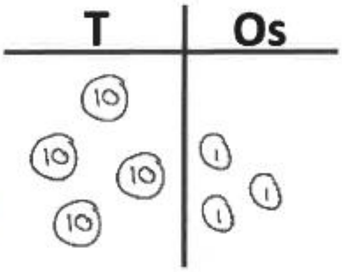


24 - 5

24 - 4 - 1 = 19

2dn - multiples of 10





43 - 20 = 23

2dn - 2dn

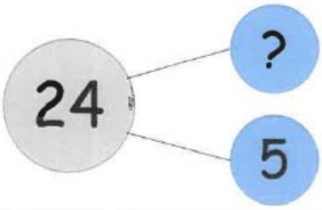
Keep the first number whole

58 - 17

58 - 10 - 7

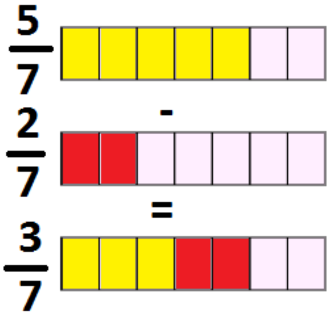
48 - 7 = 41

Children to use the part whole and bar model



24	
19	5

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$



Year 3 Objectives

- Subtracting a two digit number from another two digit number.
- Subtracting a two digit number from a three digit number.
- Subtracting fractions with the same denominator.

Key Skills:
Subtracting a one digit number from a two digit number. Subtracting multiples of ten from a two digit number. The column method.

Subtraction is not commutative

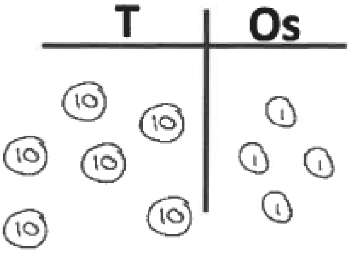
Mental calculation strategies

Count on
If the numbers are close together

203 - 199

Round and adjust
If subtracting a 'near tens' number
64 - 19

2dn - multiples of 10

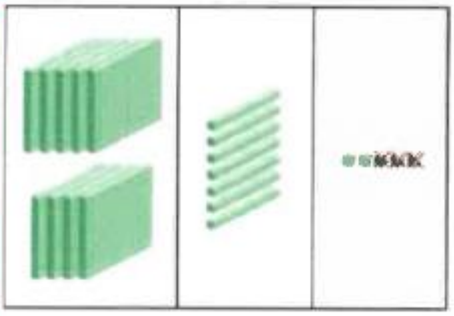


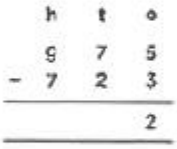
64 - 30 = 34

Count back
If subtracting a single digit or multiple of 10
342 – 5 or 257 – 40

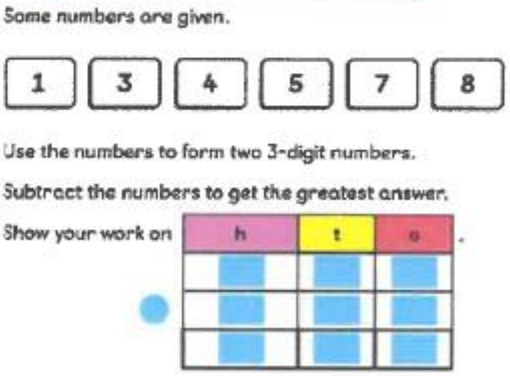
Column method

Unitise:
5 ones subtract 3 ones equals 2 ones..
7 tens subtract 2 tens equals 5 tens.
9 hundreds subtract 7 hundreds equals 2 hundreds





Problem solving with the written method



Some numbers are given.

1 3 4 5 7 8

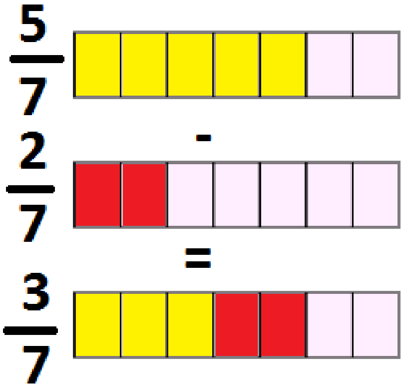
Use the numbers to form two 3-digit numbers.

Subtract the numbers to get the greatest answer.

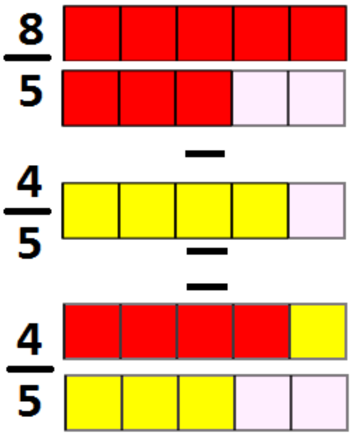
Show your work on

h	t	o

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$



$$\frac{8}{5} - \frac{4}{5} = \frac{4}{5}$$



Year 4 Objectives

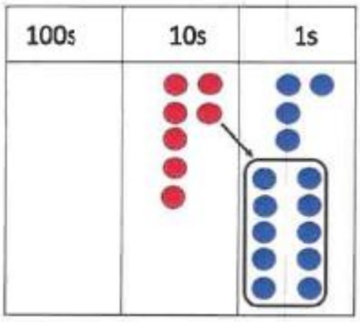
- Subtracting from a number with up to four digits.
- Constant difference.
- Subtracting fractions with the different denominator.

Key Skills:
Subtracting a one digit number from a two digit number.
Subtracting multiples of ten from a two digit number.
The column method.

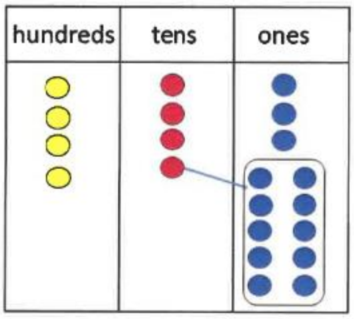
Subtraction is not commutative

Principal of constant difference

$$\begin{array}{r} \overset{6}{\cancel{7}}\,{}^{1}4 \\ -\ 27 \\ \hline 47 \end{array}$$



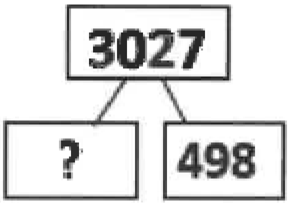
$$\begin{array}{r} 4\overset{3}{\cancel{4}}\,{}^{1}3 \\ -218 \\ \hline 225 \end{array}$$

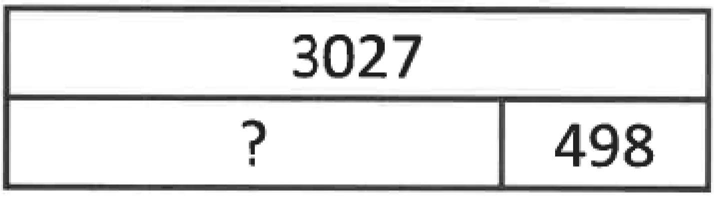


74 - 27 = 47

443 – 218 = 225

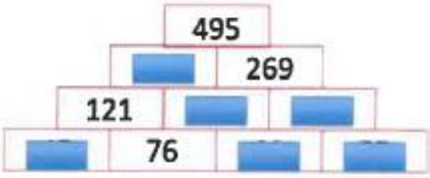
Children to use the part whole and bar model to develop estimation and number sense





Problem solving

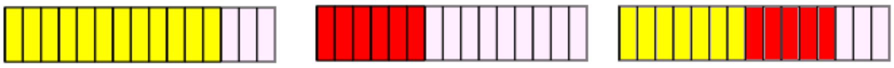
Can you complete the wall?



$$\frac{4}{5} - \frac{1}{3}$$

Convert the fractions so that the denominators are the same.

$$\frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$



Year 5 Objectives:

- Subtracting from a number with up to five digits.
- Constant difference.
- Subtraction of decimals
- Subtracting fractions with the different denominator.

Subtraction is not commutative

Principal of constant difference

- **Vary the number of digits in the number**
- **Missing boxes**
- **Balanced equations**

15.743 - 214.9 =

? – 200 = 2,307

$\frac{5}{6} - \frac{1}{4} =$

Decimal numbers

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}1\,.\,8 \\ -\ 3\,4\,.\,5 \\ \hline 3\,7\,.\,2 \\ \hline \end{array}$$

Children to use the part whole and bar model to develop estimation and number sense

375.5	
?	14.3

Problem solving

Work out whether each problem is true or false and say how he could solve the problem if it is wrong.

a) 3801 + 1499 = 3800 + 1500

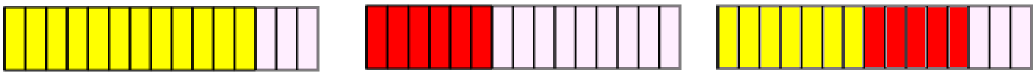
b) 3801 + 2307 = 3800 + 2310

c) 5678 – 1212 = 5670 – 1220

$$\frac{4}{5} - \frac{1}{3}$$

Convert the fractions so that the denominators are the same.

$$\frac{12}{15} - \frac{5}{15} = \frac{7}{15}$$



Year 6 Objectives

- Subtracting from a number with more than five digits.
- Subtraction of decimals.
- Subtracting fractions from a mixed number.
- Subtracting one mixed number from another.

Subtraction is not commutative

Principal of constant difference

- **Vary the number of digits in the number**
- **Missing boxes**
- **Balanced equations**

15.743 - 214.9 =

? – 200 = 2,307

$\frac{5}{6} - \frac{1}{4} =$

$$\begin{array}{r} \overset{2}{\cancel{3}}\,\overset{9}{\cancel{0}}\,\overset{{}^{1}2}{\cancel{3}}\,{}^{1}1\,.\,8 \\ -\ 1\,8\,6\,7\,.\,3 \\ \hline 1\,1\,6\,4\ .\,5 \\ \hline \end{array}$$

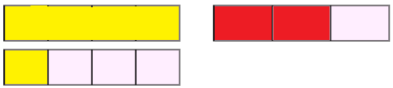
Address difficult points – zero as a place holder

$$\begin{array}{r} 57.\overset{2}{\cancel{3}}\overset{1}{\cancel{0}} \\ -\ \ 6.08 \\ \hline 51.22 \\ \hline \end{array}$$

Children to use the part whole and bar model to develop estimation and number sense

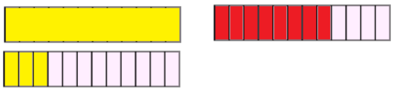
<table>
<tr><td colspan="2">487.3</td></tr>
<tr><td>?</td><td>2.9</td></tr>
</table>

$1\frac{1}{4}$ – $\frac{2}{3}$



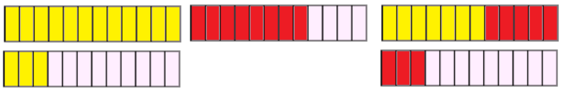
Convert the fractions so that the denominators are the same.

$1\frac{3}{12}$ – $\frac{8}{12}$

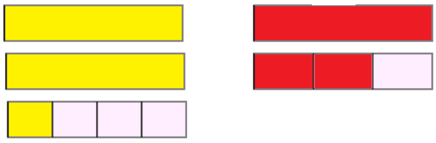


Convert the mixed number into an improper fraction.

$\frac{15}{12}$ – $\frac{8}{12}$ = $\frac{7}{12}$

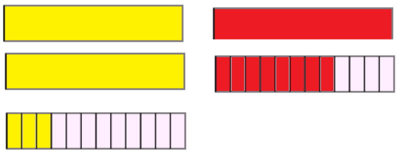


$2\frac{1}{4}$ – $1\frac{2}{3}$



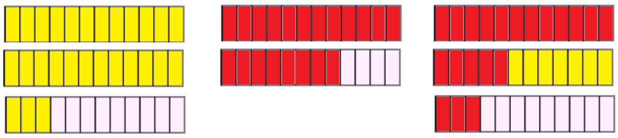
Convert the fractions so that the denominators are the same.

$2\frac{3}{12}$ – $1\frac{8}{12}$



Convert the mixed numbers into improper fractions.

$\frac{27}{12}$ – $\frac{20}{12}$ = $\frac{7}{12}$



12.0 Multiplication

Key vocabulary: double, times, multiply, multiplied by, the product of, groups of, lots of, equal groups

Subject specific vocabulary: Multiplicand x multiplier = Product multiplier x Multiplicand = Product
Factor x Factor = Multiple

Contextualise the mathematics

- WHAT DOES THIS NUMBER REPRESENT?

Expose mathematical structure and work systematically

Expect children to use correct terminology and express reasoning

- Use **STEM SENTENCES**
- Answer in **complete sentences**

Identify difficult points

- Be aware of common misconceptions
- Actively seek to uncover these

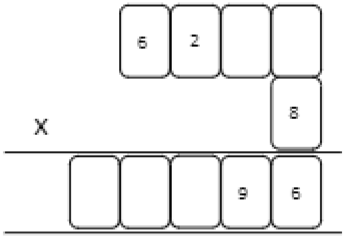
Move between the concrete, pictorial and the abstract (CPA)

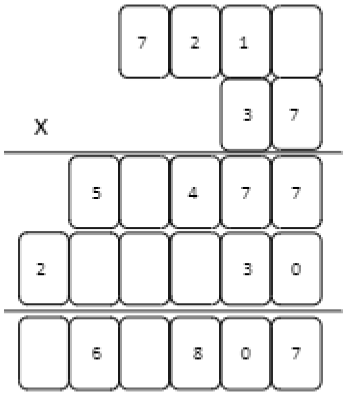
Teach inequality alongside equality

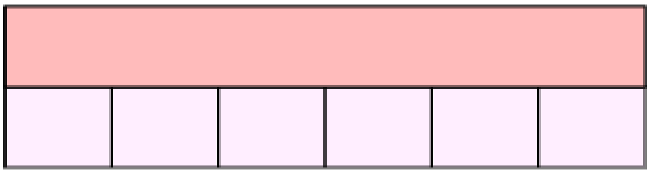
- < and > can also help deepen understanding of key concepts, eg 18p £0.15

Use empty box problems

- Promotes reasoning and finding easy ways to calculate
- Use a sequence to develop conceptual connections







12.1 The Big Ideas of Multiplication

Multiplicand x multiplier = Product   multiplier x Multiplicand = Product

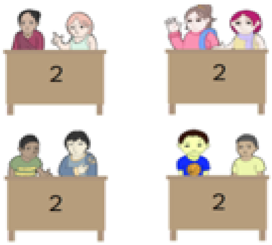
Factor x Factor = Multiple

Start by exploring unequal groups

There are some pencils. The pencils have been grouped. There are 3 groups.

There are some footballs. The footballs have been grouped. There are 3 groups.

Move from repeated addition to using the multiplication sign



2 + 2 + 2 + 2

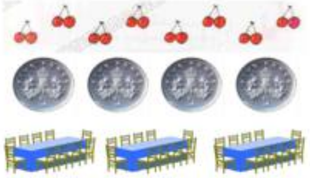
2 x 4 or 4 x 2

What does each number represent?

Move on to exploring equal groups

There are 3 groups of 5 eggs. There are 3 fives. 1 five, 2, fives, 3 fives.

- Developing fluency in counting in 2, 5 and 10:

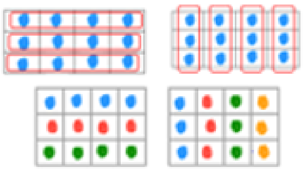


5, ____, 15, 20, ___, ___

etc

Use arrays to draw attention to the commutative structure of multiplication

4 x 3 and 3 x 4



3 x 4 = 4 x 3

12 is equal to 3 groups of 4 or 4 groups of 3

MULTIPLICATION KEY TEACHING POINTS

3 x 4   Is this 3 groups of 4 or 4 groups of 3?

At All Saints, we say: without a picture or a context to tell us which is the multiplicand and which is the multiplier, it can be either. (N.B. White Rose follows the Shanghai way of working which only allows the multiplier first, so this would be 3 groups of 4; NCETM encourages children to see this both ways so is in line with our policy.)

12.2 Multiplication Stem Sentences

A sample of Stem sentences which could be used with multiplication. This list is not complete.

factor x factor = product

When zero is a factor, the product is zero.

Multiples of 4 make equal groups of 4.

The multiplicand is the size of the group.

The multiplier is the number of groups.

Finding 10 times as many is the same as multiplying by 10 (for positive numbers);

To multiply a whole number by 10, place a zero (not add a zero) after the final digit of that number (for integers).

Finding 100 times as many is the same as multiplying by 100 (for positive numbers);

To multiply a whole number by 100, place two zeros (not add two zeros) after the final digit of that number (for integers).

12.3 Multiple Stem Sentences

A multiple of 4 is the product of 4 and a whole number. Multiples of 4 make equal groups of 4.

A multiple of a number can be divided into equal groups of that number.

A multiple of 4 can be divided into equal groups of 4.

A multiple of 4 is the product of 4 and a whole number.

12 is a multiple of 4 because you can make equal groups of 4.

13 is not a multiple of 4 because you can’t make equal groups of 4.

12.4 Factor Stem Sentences

The factors of a number are all the numbers that divide into it exactly.

A factor is number that can be divided into another number without leaving a remainder.

For example, 1, 2, 3, 4, 6 and 12 are all factors of 12.

3 is a factor of 12 because you can make 4 equal groups of 3.

4 is a factor of 12 because you can make 3 equal groups of 4.

5 is not a factor of 12 because you can’t make equal groups of 5, there will be some left over.

12.5 Prime Number Stem Sentences

A number which has only two factors is a prime number.

2 is the first, and only even, prime number.

12.6 Year Group Breakdown

Reception Objectives

- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

Key Skills:
Adding 0 and 1 to a number Addition bonds within 10 e.g. 5=4+1 Addition bonds to 10

The link between addition and multiplication can be introduced through doubling.

If available, Numicon is used to visualise the repeated adding of the same number.
These can then be drawn around or printed as a way of recording.

Children begin with mostly pictorial representations:

How many groups of 2 are there?

Real life contexts and use of practical equipment to count in repeated groups of the same size:

How many wheels are there altogether?

How much money do I have?

Count in twos; fives; tens both aloud and with  objects

Children are given multiplication problems set in a real life context. Children are encouraged to visualise the problem.

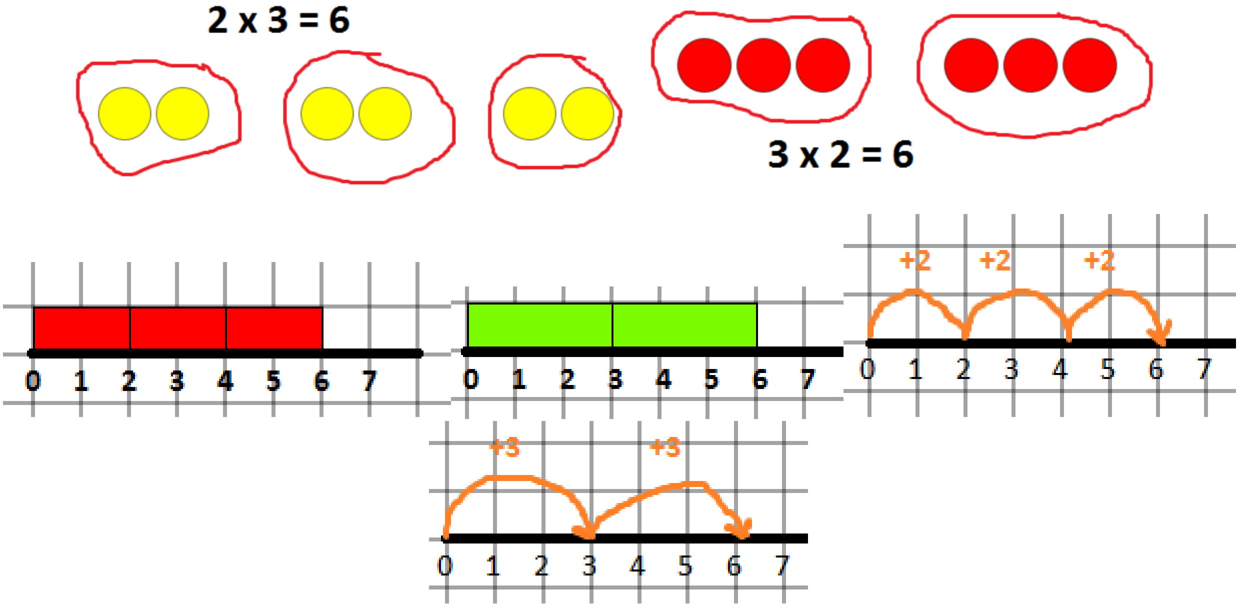
How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?

Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two"

Year 1 Objectives

- Multiplication through repeated addition.

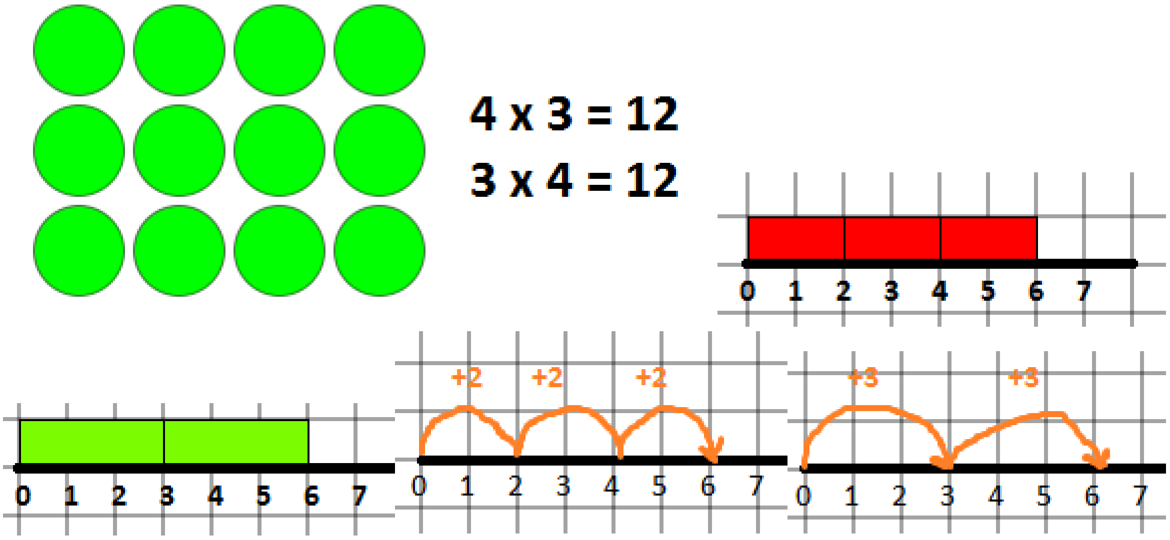
Key Skills:
Adding 0 and 1 to a number
Addition bonds within 10 e.g. 5=4+1
Addition bonds to 10



Year 2 Objectives:

- 2, 5 and 10s multiplication facts.
- Multiplication as repeated addition.
- Multiplication through arrays.

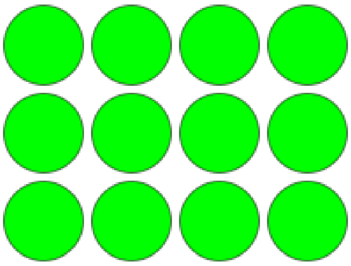
Key Skills:
Adding 0 and 1 to a number
Addition bonds within 20 e.g. 5=4+1
Addition bonds to 20



Year 3 Objectives

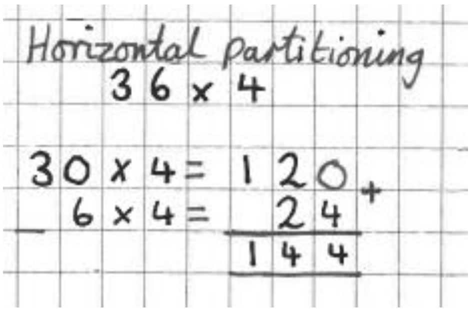
- 2, 3, 4, 5, 6, 8 and 10s multiplication facts.
- Two-digit numbers multiplied by one-digit numbers.
- Multiplication by arrays.
- Column method for multiplication.

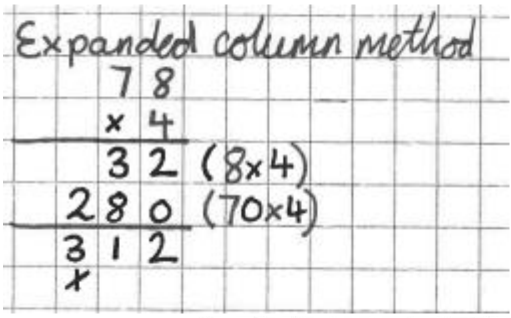
Key Skills:
2, 3, 4, 5, 6, 8 and 10s multiplication facts. Multiplying a number by 10.

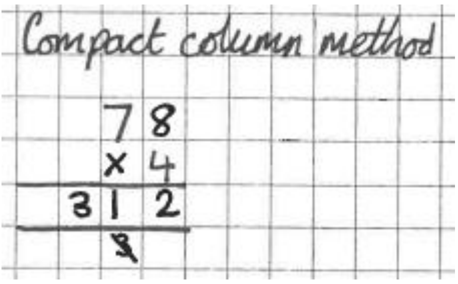


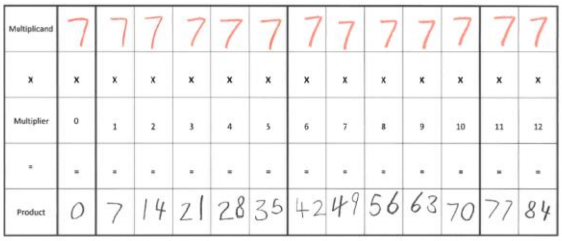
4 x 3 = 12

3 x 4 = 12

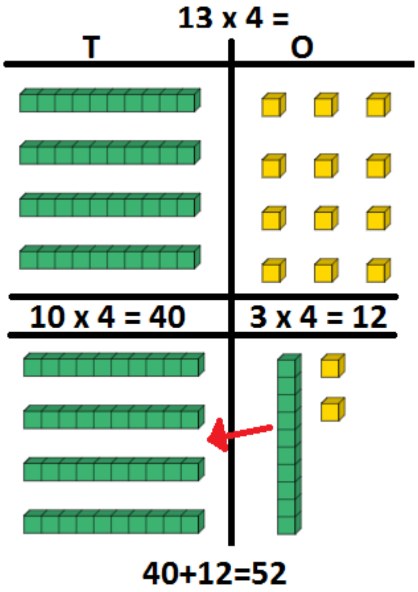


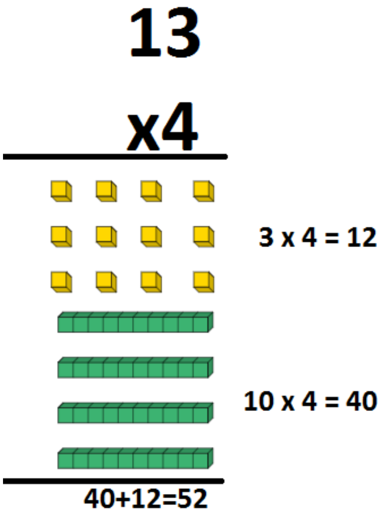


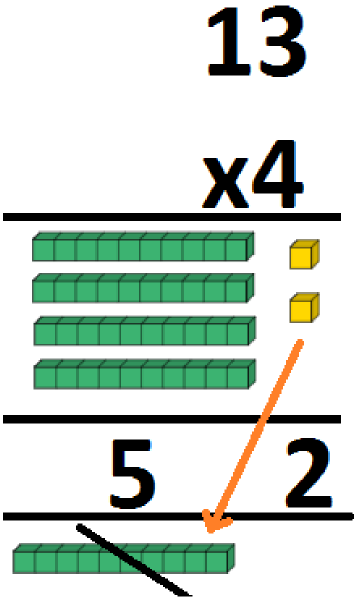


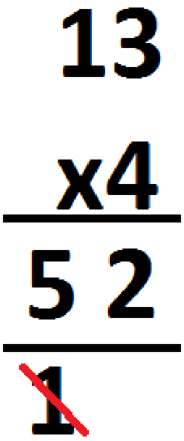


Multiplicand	7	7	7	7	7	7	7	7	7	7	7	7	7
x	x	x	x	x	x	x	x	x	x	x	x	x	x
Multiplier	0	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Product	0	7	14	21	28	35	42	49	56	63	70	77	84





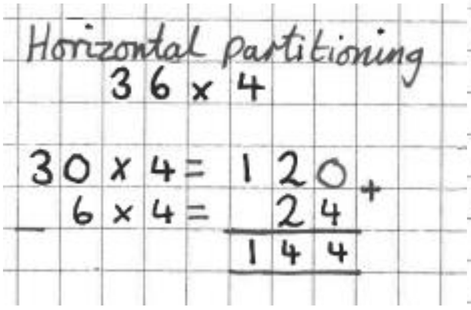




Year 4 Objectives

- Multiplication facts up to 12 x 12.
- Two-digit numbers multiplied by one-digit numbers.
- Two-digit numbers multiplied by two-digit numbers.
- Column method for multiplication.
- Multiplying a fraction by another fraction.

Key Skills:
Multiplication facts up to 12x12 Multiplying a number by 10 and 100



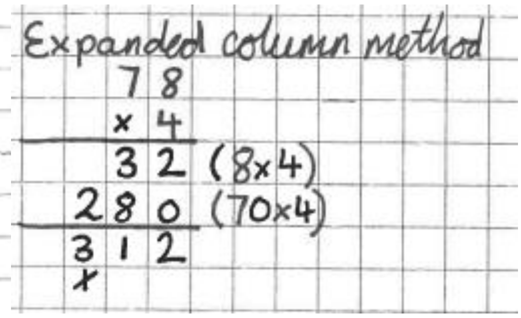
Horizontal partitioning

36 x 4

30 x 4 = 120 +

6 x 4 = 24

144



Expanded column method

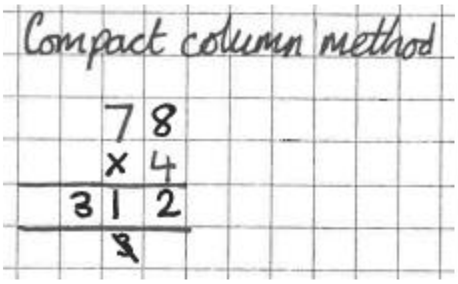
78

x 4

32 (8x4)

280 (70x4)

312



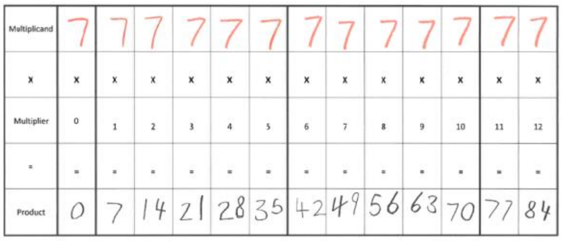
Compact column method

78

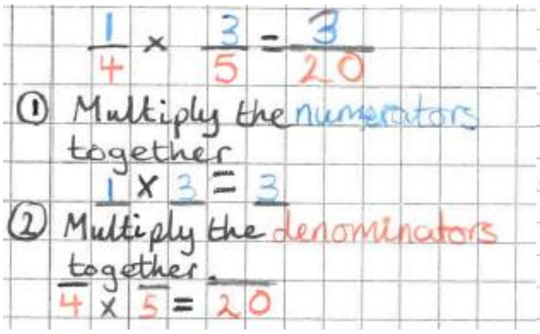
x 4

312

3



Multiplicand	7	7	7	7	7	7	7	7	7	7	7	7	7
x	x	x	x	x	x	x	x	x	x	x	x	x	x
Multiplier	0	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Product	0	7	14	21	28	35	42	49	56	63	70	77	84



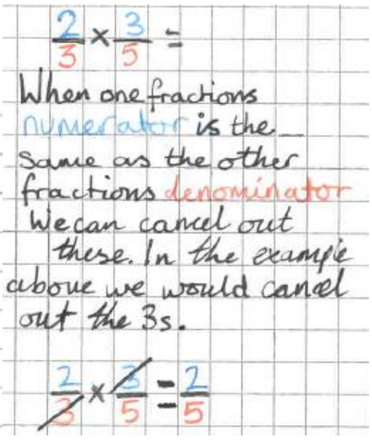
$\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

① Multiply the numerators together

1 x 3 = 3

② Multiply the denominators together

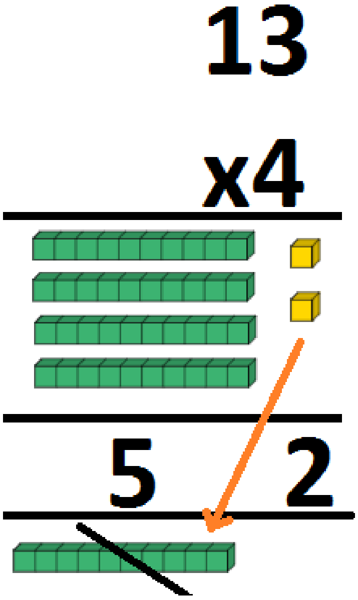
4 x 5 = 20

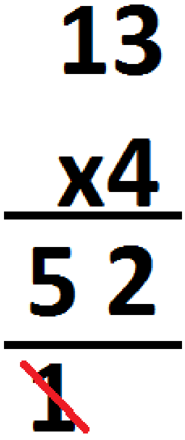


$\frac{2}{3} \times \frac{3}{5} =$

When one fractions numerator is the same as the other fractions denominator We can cancel out these. In the example above we would cancel out the 3s.

$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}$

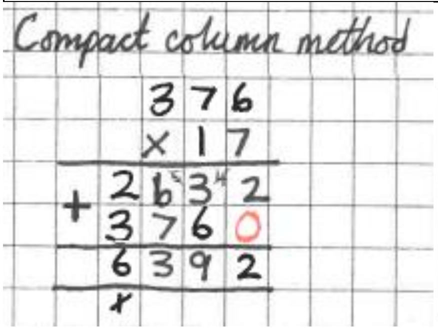




Year 5 Objectives:

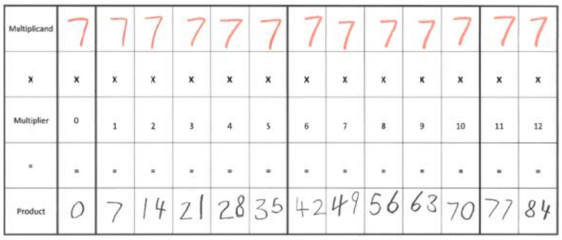
- Multiplication facts up to 12 x 12.
- One-digit numbers multiplied by two one-digit numbers.
- Three-digit numbers multiplied by two-digit numbers.
- Four-digit numbers multiplied by two-digit numbers.
- Column method for multiplication.
- Multiplying a fraction by another fraction.
- Multiplying a fraction by a whole number.

Key Skills:
Multiplication facts up to 12x12 Multiplying a number by 10, 100 and 1,000.

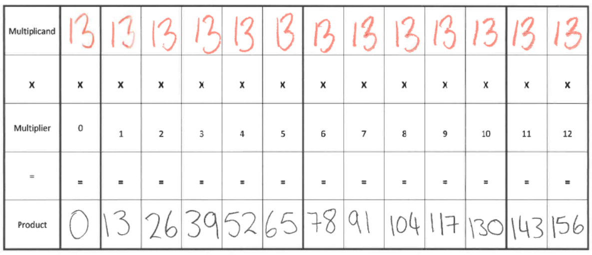


Compact column method

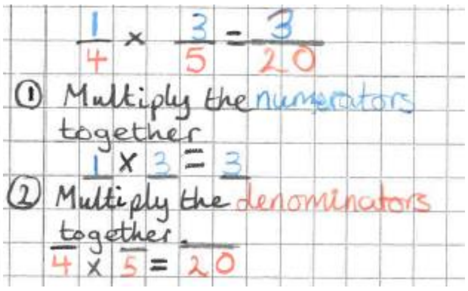
$$\begin{array}{r} 3\,7\,6 \\ \times\ 1\,7 \\ \hline 2\,6^{5}3^{4}2 \\ +\ 3\,7\,6\,0 \\ \hline 6\,3\,9\,2 \\ \hline 1 \end{array}$$



Multiplicand	7	7	7	7	7	7	7	7	7	7	7	7	7
x	x	x	x	x	x	x	x	x	x	x	x	x	x
Multiplier	0	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Product	0	7	14	21	28	35	42	49	56	63	70	77	84



Multiplicand	13	13	13	13	13	13	13	13	13	13	13	13	13
x	x	x	x	x	x	x	x	x	x	x	x	x	x
Multiplier	0	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Product	0	13	26	39	52	65	78	91	104	117	130	143	156



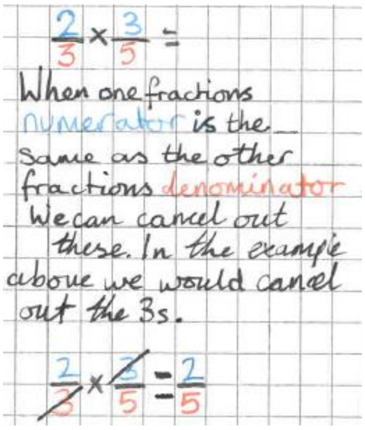
$$\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$$

① Multiply the numerators together

$$1 \times 3 = 3$$

② Multiply the denominators together.

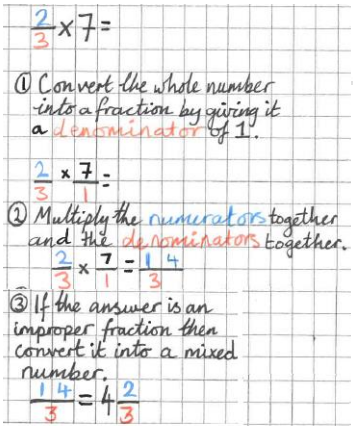
$$4 \times 5 = 20$$



$$\frac{2}{3} \times \frac{3}{5} =$$

When one fractions numerator is the same as the other fractions denominator We can cancel out these. In the example above we would cancel out the 3s.

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}$$



$$\frac{2}{3} \times 7 =$$

① Convert the whole number into a fraction by giving it a denominator of 1.

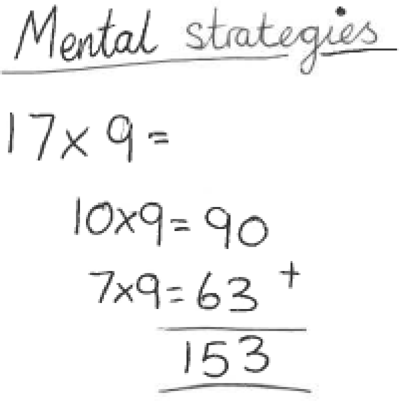
$$\frac{2}{3} \times \frac{7}{1} =$$

② Multiply the numerators together and the denominators together.

$$\frac{2}{3} \times \frac{7}{1} = \frac{14}{3}$$

③ If the answer is an improper fraction then convert it into a mixed number.

$$\frac{14}{3} = 4\frac{2}{3}$$



Mental strategies

17x9=

10x9=90

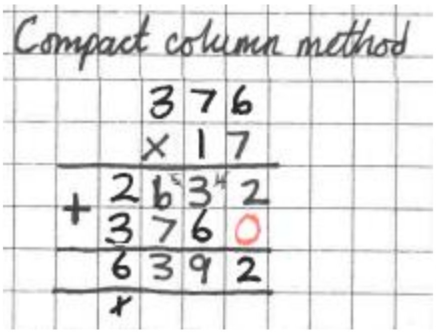
7x9=63 +

153

Year 6 Objectives

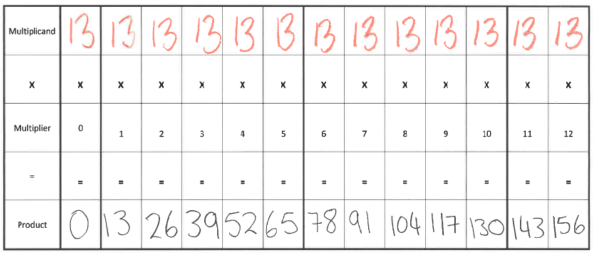
- Multiplication facts up to 12 x 12.
- One-digit numbers multiplied by 2 one-digit numbers.
- Three-digit numbers multiplied by two-digit numbers.
- Four-digit numbers multiplied by two-digit numbers.
- Multiplying decimals.
- Column method for multiplication.
- Multiplying a fraction by another fraction.
- Multiplying a fraction by a whole number.
- Multiplying a mixed number by another fraction.
- Multiplying a mixed number by another mixed number.

Key Skills:
Multiplication facts up to 12x12 Multiplying a number by multiples of 10, 100 and 1,000.

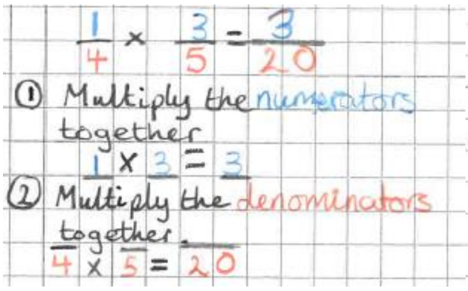


Compact column method

$$\begin{array}{r} 376 \\ \times\ 17 \\ \hline 2\,6^{5}3^{4}2 \\ +\ 3760 \\ \hline 6392 \\ \hline \end{array}$$



Multiplicand	13	13	13	13	13	13	13	13	13	13	13	13	13
x	x	x	x	x	x	x	x	x	x	x	x	x	x
Multiplier	0	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Product	0	13	26	39	52	65	78	91	104	117	130	143	156



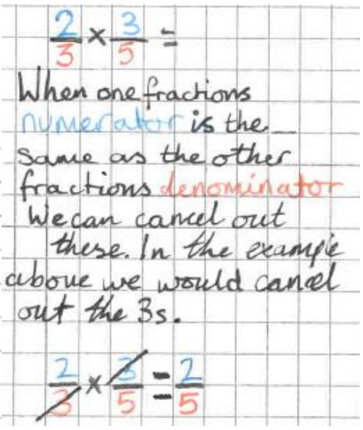
$\frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

① Multiply the numerators together

$1 \times 3 = 3$

② Multiply the denominators together

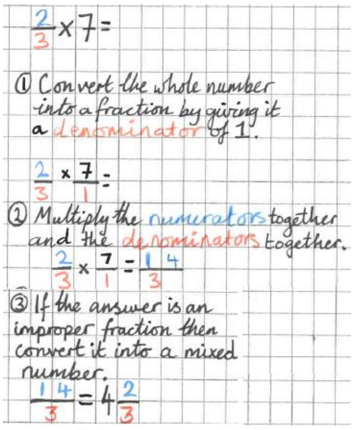
$4 \times 5 = 20$



$\frac{2}{3} \times \frac{3}{5} =$

When one fractions numerator is the same as the other fractions denominator We can cancel out these. In the example above we would cancel out the 3s.

$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} = \frac{2}{5}$



$\frac{2}{3} \times 7 =$

① Convert the whole number into a fraction by giving it a denominator of 1.

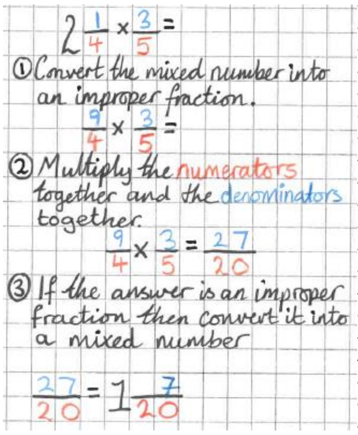
$\frac{2}{3} \times \frac{7}{1} =$

② Multiply the numerators together and the denominators together.

$\frac{2}{3} \times \frac{7}{1} = \frac{14}{3}$

③ If the answer is an improper fraction then convert it into a mixed number.

$\frac{14}{3} = 4\frac{2}{3}$



$2\frac{1}{4} \times \frac{3}{5} =$

① Convert the mixed number into an improper fraction.

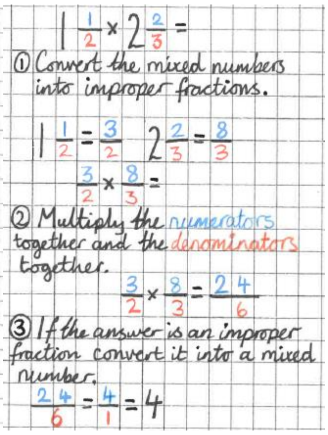
$\frac{9}{4} \times \frac{3}{5} =$

② Multiply the numerators together and the denominators together.

$\frac{9}{4} \times \frac{3}{5} = \frac{27}{20}$

③ If the answer is an improper fraction then convert it into a mixed number

$\frac{27}{20} = 1\frac{7}{20}$



$1\frac{1}{2} \times 2\frac{2}{3} =$

① Convert the mixed numbers into improper fractions.

$1\frac{1}{2} = \frac{3}{2}$ $2\frac{2}{3} = \frac{8}{3}$

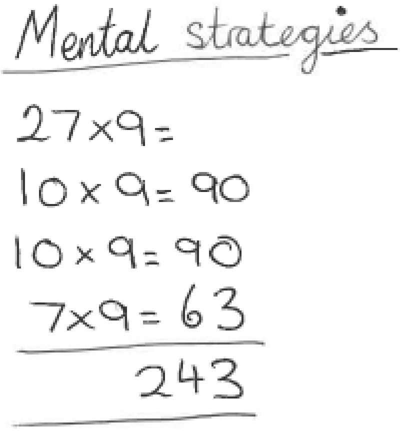
$\frac{3}{2} \times \frac{8}{3} =$

② Multiply the numerators together and the denominators together.

$\frac{3}{2} \times \frac{8}{3} = \frac{24}{6}$

③ If the answer is an improper fraction convert it into a mixed number.

$\frac{24}{6} = \frac{4}{1} = 4$



Mental strategies

$27 \times 9 =$

$10 \times 9 = 90$

$10 \times 9 = 90$

$7 \times 9 = 63$

243

13.0 Division

Key vocabulary: shared equally, equal groups, divide, divide by, half

Subject specific vocabulary: Dividend ÷ Divisor = Quotient

Contextualise the mathematics
- **WHAT DOES THIS NUMBER REPRESENT?**

Expose mathematical structure and work systematically

Expect children to use correct terminology and express reasoning
- Use **STEM SENTENCES**
- Answer in **complete sentences**

Identify difficult points
- Be aware of common misconceptions
- Actively seek to uncover these

Move between the concrete, pictorial and the abstract (CPA)

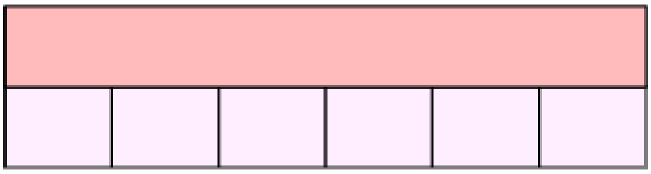
Teach inequality alongside equality

- < and > can also help deepen understanding of key concepts, eg 18p £0.15

Use empty box problems
- Promotes reasoning and finding easy ways to calculate
- Use a sequence to develop conceptual connections

	1	3		9	
6		1	5	4	

	1	9	7	6	
5	9			0	



13.1 The Big Ideas of Division

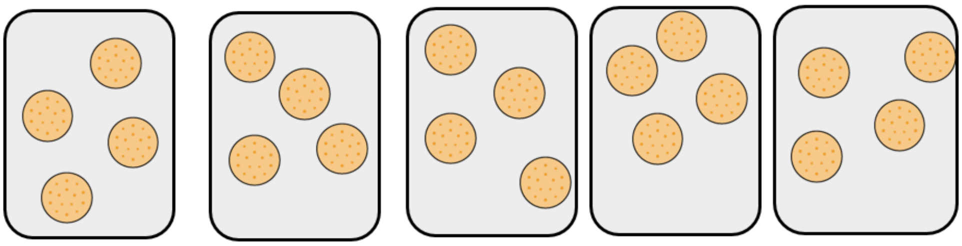
Dividend ÷ Divisor = Quotient

There are two structures of division: Quotitive and Partitive.

Quotitive Structure:

Division equations can be used to represent ‘grouping’ problems. This is called quotitive structure.

Quotitive (grouping) structure of division



20 is divided into groups of 4. There are 5 groups.

20 ÷ 4 = 5

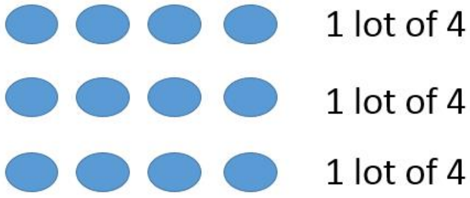
20 divided into groups of 4 is equal to 5.

Partitive structure:

Division equations can be used to represent ‘sharing’ problems. This is called partitive structure.

Partitive (sharing) structure of division

12 chicken nuggets are shared equally between 4 children. How many for each child?



12 divided between 4 is equal to 3 nuggets each. 12 ÷ 4 = 3.

We think about how many of the divisor fit into the dividend.

12 ÷ 4 How many ‘4’s are there in 12.

Objects can be grouped equally, sometimes with a remainder.

Division is not commutative. We start with the whole and think about how many equal parts there are in the whole.

13.2 Division Stem Sentences

Dividend divided by the divisor equals the quotient.

We can use our multiplication facts to help us with division

12 ÷ 4 How many '4's are there in 12? 3 x 4 = 12

When we divide into groups, the divisor is kept as a group.

When we divide by sharing, the divisor is partitioned.

When the dividend is zero, the quotient is zero;

When the dividend is equal to the divisor, the quotient is one;

When the divisor is equal to one, the quotient is equal to the dividend

We should never write a calculation where the divisor is zero.

The dividend in ________, the divisor is __________. The quotient is __________.

13.3 Multiple Stem Sentences

A multiple of 4 is the product of 4 and a whole number. Multiples of 4 make equal groups of 4.

A multiple of a number can be divided into equal groups of that number.

A multiple of 4 can be divided into equal groups of 4.

A multiple of 4 is the product of 4 and a whole number.

12 is a multiple of 4 because you can make equal groups of 4.

13 is not a multiple of 4 because you can't make equal groups of 4.

13.4 Factor Stem Sentences

The factors of a number are all the numbers that divide into it exactly.

A factor is number that can be divided into another number without leaving a remainder.

For example, 1, 2, 3, 4, 6 and 12 are all factors of 12.

3 is a factor of 12 because you can make 4 equal groups of 3.

4 is a factor of 12 because you can make 3 equal groups of 4.

5 is not a factor of 12 because you can't make equal groups of 5, there will be some left over.

13.5 Prime Number Stem Sentences

A number which has only two factors is a prime number.

2 is the first, and only even, prime number.

13.6 Year Group Breakdown

Reception Objectives

- They solve problems, including doubling, halving and sharing.

Key Skills:
Adding 0 and 1 to a number Addition bonds within 10 e.g. 5=4+1 Addition bonds to 10

The ELG states that children solve problems, including doubling, halving and sharing. Children need to see and hear representations of division both as equal grouping and equal sharing.

Division can be introduced through halving.

Children begin with mostly pictorial representations linked to real world contexts:

Grouping model

Mum has 6 socks. She grouped them into pairs – how many pairs did she make?

Sharing model

I have 10 sweets. I want to share them with my friend. How many will we have each?

"I have got 5 bones to share between my two dogs. How many bones will they get each?"

Children have a go at recording the calculation that has been carried out.

2 ½ + 2 ½ = 5

Year 1 Objectives

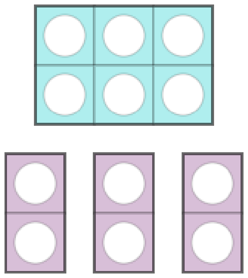
- Grouping and sharing objects and pictures.

Key Skills:
Adding 0 and 1 to a number Addition and subtraction bonds within 10. Addition and subtraction bonds to 10.

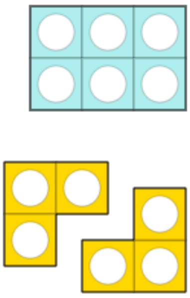
Link to fractions

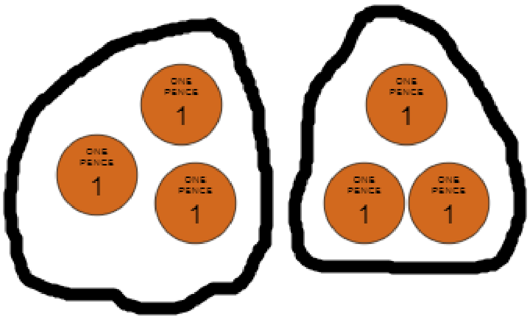
$\frac{1}{2}$ of ____ and ÷ 2

6 ÷ 3 = 2



6 ÷ 2 = 3





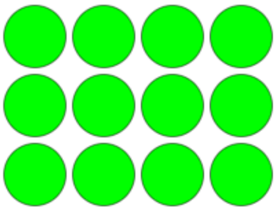
Year 2 Objectives:

- 2, 4, 5 and 10s division facts.
- Grouping and sharing.
- Repeated subtraction.
- Two-digit numbers divided by one-digit numbers.
- Word problems including division.

Key Skills:
Addition and subtraction bonds within 10. Addition and subtraction bonds to 10. 2, 4, 5 and 10 multiplication and division facts.

Link to fractions

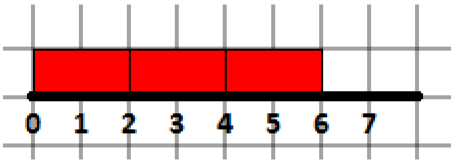
$\frac{1}{2}$ of ____ and ÷ 2

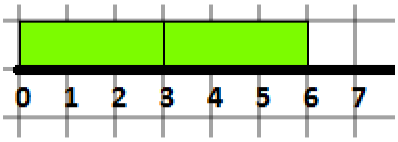


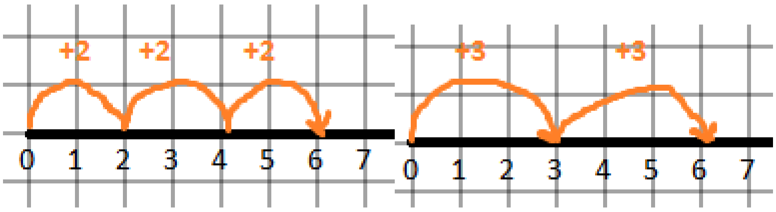
Arrays are useful to show how dividion and multiplication are linked.

12 ÷ 4 = 3 12 ÷ 3 = 4

Bar models also show the connections between division and multiplication.







Number lines are good for showing grouping.

Year 3 Objectives

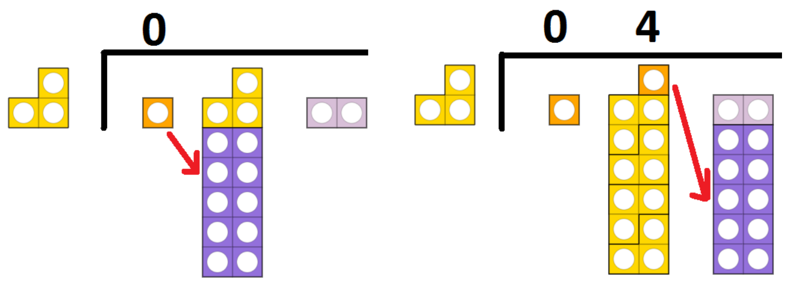
- 2, 3, 4, 5, 6, 8 and 10 division facts.
- Grouping and sharing.
- Two-digit numbers divided by one-digit numbers.
- Quotients with remainders (r).

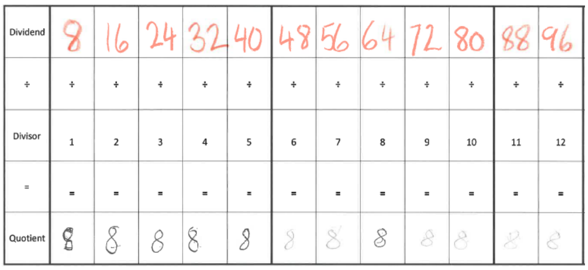
Key Skills:

Addition and subtraction bonds within 20.
Addition and subtraction bonds to 20.
2, 3, 4, 5, 6, 8 and 10 multiplication and division facts.

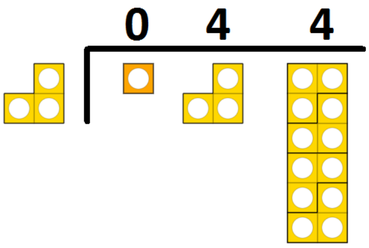
Link to fractions

$\frac{1}{2}$ of ____ and ÷ 2



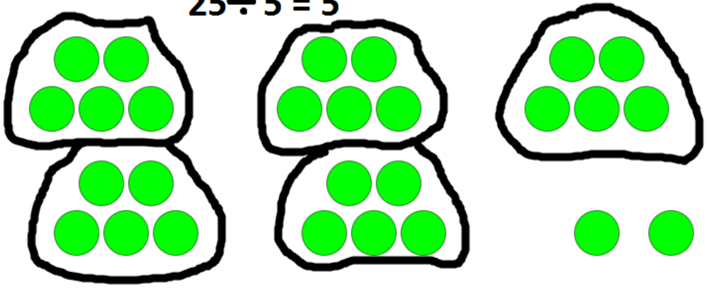


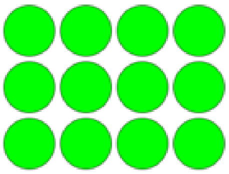
Dividend	8	16	24	32	40	48	56	64	72	80	88	96
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷
Divisor	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=
Quotient	8	8	8	8	8	8	8	8	8	8	8	8



$$\begin{array}{r} 0\ \ 4\ \ 4 \\ 3\overline{)1\ {}^{1}3\ {}^{1}2} \end{array}$$

25÷5 = 5 r2

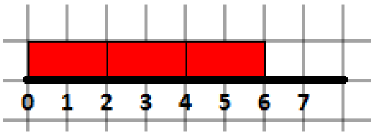


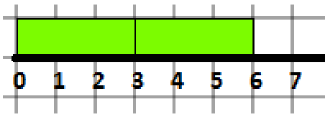


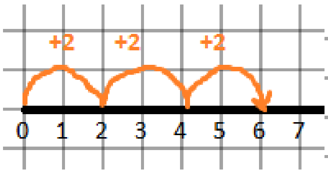
Arrays are useful to show how dividion and multiplication are linked.

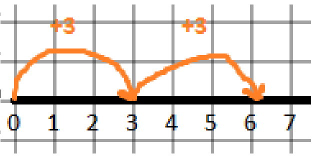
12 ÷ 4 = 3   12 ÷ 3 = 4

Bar models also show the connections between division and multiplication.









Number lines are good for showing grouping.

Year 4 Objectives

- Division facts up to 144÷12.
- Three-digit numbers divided by one-digit numbers.
- Quotients with remainders (r).
- ‘Bus stop’ method.
- Dividing a fraction by another fraction.

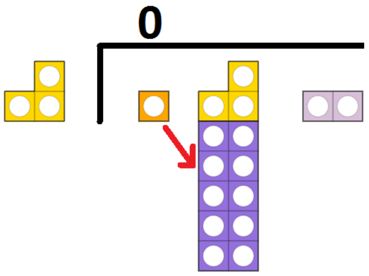
Key Skills:
Multiplication facts up to 12x12. Division facts up to 144÷12. Addition and Subtraction bonds within 20. Addition and subtraction bonds to 20.

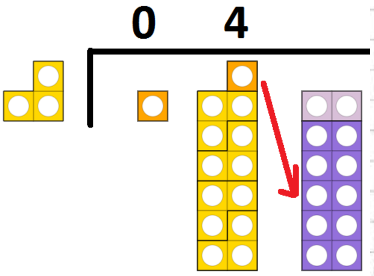
Link to fractions

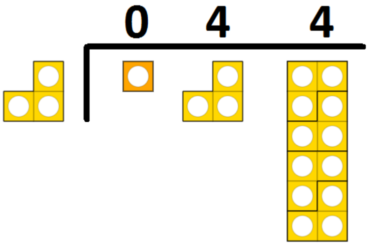
$\frac{1}{2}$ of ____ and ÷ 2

Dividend	8	16	24	32	40	48	56	64	72	80	88	96
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷
Divisor	1	2	3	4	5	6	7	8	9	10	11	12
=	=	=	=	=	=	=	=	=	=	=	=	=
Quotient	8	8	8	8	8	8	8	8	8	8	8	8

Dividend	12	18	18	48	45	42	49	56	54	100	121	132	602
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷
Divisor	1	2	3	4	5	6	7	8	9	10	11	12	100
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Quotient	12	9	6	12	9	7	7	7	6	10	11	11	6.02







$$\begin{array}{r|lll} & 0 & 4 & 4\\ \hline 3 & 1 & {}^{1}3 & {}^{1}2\end{array}$$

$$\begin{array}{r|lll} & 8 & 6 & r2\\ \hline 5 & 4 & 3^{3} & 2\end{array}$$

$\frac{3}{4} \div \frac{2}{5}$

① Keep the first fraction the same.
Change the divide sign to a multiply sign.
Flip the numerator and denominator of the second fraction

$\frac{3}{4} \times \frac{5}{2}$

② Multiply the numerators together and the denominators together.

$\frac{3}{4} \times \frac{5}{2} = \frac{15}{12}$

③ If your answer is an improper fraction, convert it into a mixed number and simplify.

$\frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$

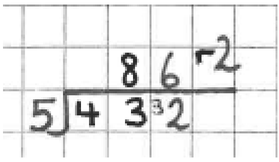
Year 5 Objectives:

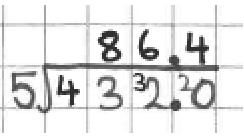
- Division facts up to 144÷12.
- Three-digit numbers divided by one-digit numbers.
- Four-digit numbers divided by one-digit numbers.
- Quotients with remainders as a fraction or a decimal.
- 'Bus stop' method.
- Dividing a fraction by another fraction.
- Dividing a fraction by a whole number.

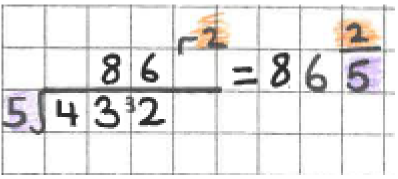
Key Skills:
Multiplication facts up to 12x12. Division facts up to 144÷12. Addition and Subtraction bonds within 20. Addition and subtraction bonds to 20.

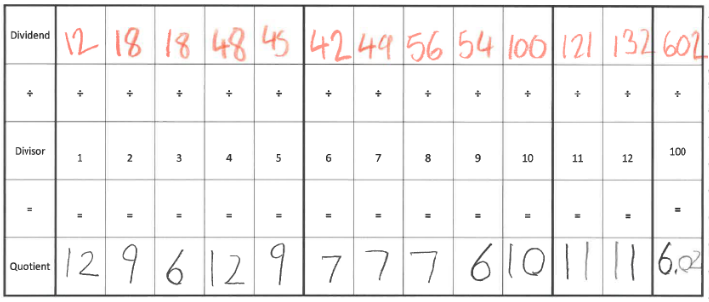
Link to fractions

$\frac{1}{2}$ of ____ and ÷ 2

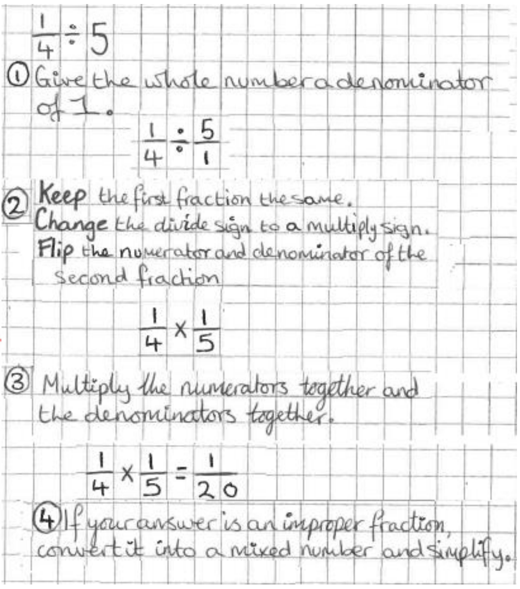








Dividend	12	18	18	48	45	42	49	56	54	100	121	132	602
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷
Divisor	1	2	3	4	5	6	7	8	9	10	11	12	100
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Quotient	12	9	6	12	9	7	7	7	6	10	11	11	6.02



$\frac{1}{4} \div 5$

① Give the whole number a denominator of 1.

$\frac{1}{4} \div \frac{5}{1}$

② **Keep** the first fraction the same.
Change the divide sign to a multiply sign.
Flip the numerator and denominator of the second fraction.

$\frac{1}{4} \times \frac{1}{5}$

③ Multiply the numerators together and the denominators together.

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

④ If your answer is an improper fraction, convert it into a mixed number and simplify.

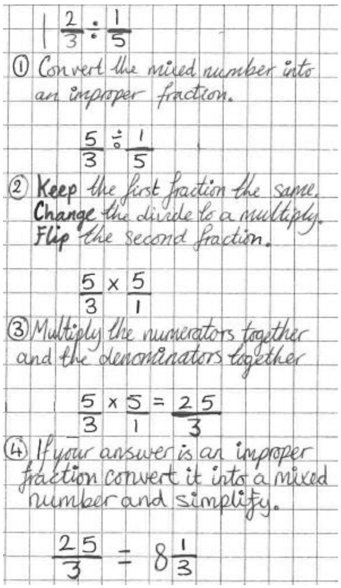
Year 6 Objectives

- Division facts up to 144÷12.
- Four-digit numbers divided by one-digit numbers.
- Four-digit numbers divided by two-digit numbers.
- Quotients with remainders as a fraction or a decimal.
- ‘Bus stop’ method.
- Dividing a fraction by another fraction.
- Dividing a fraction by a whole number.
- Dividing a mixed number by a fraction.

Key Skills:
Multiplication facts up to 12x12. Division facts up to 144÷12. Addition and Subtraction bonds within 20. Addition and subtraction bonds to 20.

Link to fractions

$\frac{1}{2}$ of ____ and ÷ 2



$1\frac{2}{3} \div \frac{1}{5}$

① Convert the mixed number into an improper fraction.

$\frac{5}{3} \div \frac{1}{5}$

② **Keep** the first fraction the same. **Change** the divide to a multiply. **Flip** the second fraction.

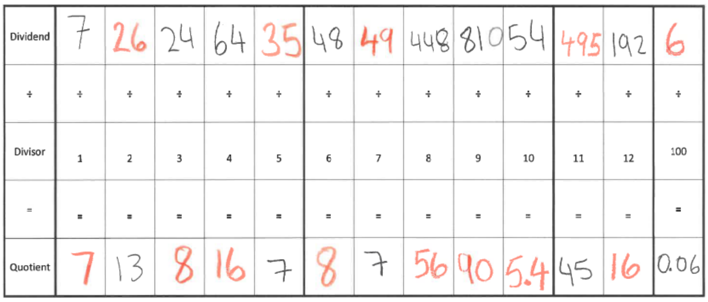
$\frac{5}{3} \times \frac{5}{1}$

③ Multiply the numerators together and the denominators together

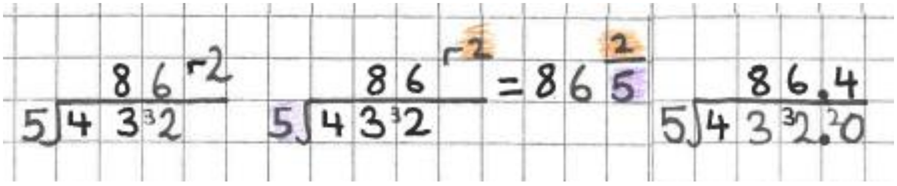
$\frac{5}{3} \times \frac{5}{1} = \frac{25}{3}$

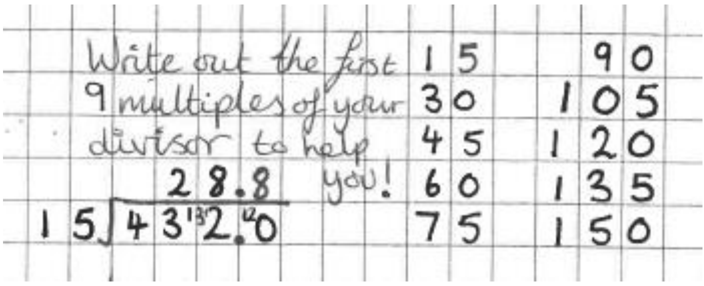
④ If your answer is an improper fraction convert it into a mixed number and simplify.

$\frac{25}{3} = 8\frac{1}{3}$

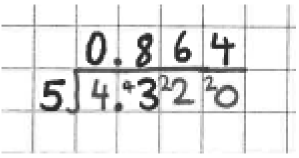


Dividend	7	26	24	64	35	48	49	448	810	54	495	192	6
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	÷
Divisor	1	2	3	4	5	6	7	8	9	10	11	12	100
=	=	=	=	=	=	=	=	=	=	=	=	=	=
Quotient	7	13	8	16	7	8	7	56	90	5.4	45	16	0.06





Write out the first 9 multiples of your divisor to help you!



14.0 Staff Induction

As part of their induction, new staff members will be made familiar with this policy.

15.0 Linked Policies

This policy and procedures should be read in conjunction with other related policies, including:

- Mathematics Policy.
- Parents’ Guide to Calculation.

1.0 Identifying Misconceptions

1.1 Addition

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Carry out column addition of numbers involving decimal places.**	Unless a pupil has a good understanding of place value they will continue to make mistakes with column addition. Such errors are often dismissed as careless mistakes, when the pupil in fact has a fundamental weakness in their understanding. When adding with decimals such weaknesses are highlighted because of the 'decimal point'.	What is each digit worth? Which is the tenths column? Which is the hundredths? Which are correct/ incorrect? How do you know?	Estimation – Pupils must learn to estimate – This way they will know when they have made an error. Add numbers to one and then two decimal places to begin with. Use the example of money to teach the concept e.g. £3.12 + £4.15 Then extend so that a 'carry' is required. Give the children some completed questions to mark. All questions need to be written horizontally as well as in column form. Include incorrect answers.

5	**Carry out column addition of positive integers less than 10,000.**	As numbers get larger, pupils miscalculate because of a lack of understanding of the place value of numbers. e.g. 1163 + 12123 23753 Some pupils will not realise that they will have to add a ‘carried’ number.	Why is it important to place the digits in the correct columns? What tips would you give someone to help them with column addition?	Estimation

4	**Use known number facts and place value to add mentally, including any pair of two-digit whole numbers.** **Carry out column addition of two integers less than 1000, and column addition of more than two such integers.**	Pupils sometimes begin adding with the left hand column first. Not understanding the concept of a ‘carry’ when a number totals more than ten, hundred etc. e.g. 99 + 101 1910 Pupils find it difficult to add when a zero is involved. They might not record a zero in an answer, leading to the following situation: 103 + 406 59	What strategies would you use to work out the answers to these calculations? Could you use a different method? Which column do you begin with? Does your answer make sense? What tips would you give someone to help them with column addition? Zero is a placeholder. What does this mean? Why is it important to include zeros in our answers?	These pupils could carry out more examples using Base Ten pieces and then linking each practical step to a recorded step. Estimate the answer and check that their answer is similar to the estimation. They should realise that it means adding ‘nothing’. When they have an answer of zero, they often need to be reminded to record it.

3	**Know by heart all addition facts for each number to 20.** **Add mentally a ‘near multiple of 10’ to a two-digit number.**	Pupils are sometimes confused that addition is associative i.e. 3+1=4 and 1+3=4. If they were to understand this concept they would find it much easier to recall the addition facts. If teachers use the phrase ‘near multiple of ten’ the children are often confused and believe that they should be multiplying a number. If they understand the term correctly then they might still struggle with compensating, not knowing whether to add or subtract. E.g. 46+19 = 46+20 -1 often confused as 46-20 + 1	How many different ways can you make 10 by adding two numbers together? Did you find a quick way to work out the answer? What do we mean when we say ‘a near multiple of 10?’ Why is this a good method? Show me on a 100 square.	Challenge – How many ways can you make 20 by adding 3 numbers together? Demonstrate the associative rule in order to make their working more efficient. Demonstrate what is happening on a number line. Use simpler terms to describe the operation e.g. ‘add ten and take one away’.

2	**Know by heart all addition facts for each number to at least 10.**		Look at this number sentence _+_=7. What could the two missing numbers be?	
	Use knowledge that addition can be done in any order to do mental calculations more efficiently.	Pupils believe that they have to add in the order that the question was asked in.	I want to find the total of 2, 14 and 8? Tell me some different ways that we could add them. Would it be better to begin adding with the largest number first, or two numbers that can make 10?	Chain sums – Children to close their eyes. Give them 3 or 4 numbers to add together. Who is the fastest? What is the best way to add them? Why?

1	**Understand the operation of addition and use the related vocabulary.** **Use mental strategies to solve simple problems using addition, explaining methods and reasoning orally.** **Know by heart all number pairs with a total of 10.**	__ + 3 = 10 When pupils are faced with problems such as the above they see two numbers and add them (e.g. 3+10=13) instead of reading it as a sentence. Pupils 'count on' to find the difference between their starting number and ten. Sometimes they are unsure of number order and therefore make mistakes. Sometimes they count their starting number e.g. when finding the number pair 6 + - = 10 they begin counting with the six and say '6, 7, 8, 9, 10' and therefore believe the missing number to be 5.	What is the missing number>? What do you need to add to 3 to get to 10? Which number should you start with?	Using blank number lines to enable pupils to visualise the sentence. Practise counting. Encourage children to tap their heads when saying their starting number and them count on their fingers for the following numbers. e.g. '6 (tap head), 7, 8, 9, 10 (count on fingers). They will then get the correct answer, 4.

R	**Begin to relate addition to combining two groups of objects.** **In practical activities and discussion, begin to use the vocabulary involved in adding.**	Not being able to ‘hold’ the number that they started with, when adding the second group. Not knowing the number order when counting on.	Which number are you going to start with? The smallest or the largest? Questions relating to number order such as: Is 7 larger or smaller than 5? What comes after 4?	Using objects and visual aids whilst counting on and/or combining groups. Children to place numbers (or groups of objects) in an order and encourage them to say which is the smallest or the largest. Using a playhouse, start with 3 people in one room and four in another – ‘which room has more people in it? How do you know? Move some people from one room to another – What has happened in this room?

1.2 Subtraction

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Carry out column subtraction of numbers involving decimals.**	Subtractions involving zeros cannot be done. That calculations such as the following cannot be done: 34 – 27 Pupils who cannot do these have not got a sufficient understanding of exchanging. Unless a pupil has a good understanding of place value they will continue to make mistakes with column subtraction. Such errors are often dismissed as careless mistakes, when the pupil in fact has a fundamental weakness in their understanding. When subtracting with decimals such weaknesses are highlighted because of the 'decimal point'.	Can a subtraction such as 203 - 132 be done? How can we take 30 from 0? If you think that this can't be done using a written method can it be done mentally? If it can be done mentally then surely it can be done using a written method. What is each digit worth? Which is the tenths column? Which is the hundredths? Which are correct/ incorrect? How do you know?	Revise decomposition. If necessary, reinforce the method using base ten materials on an OHP or by using a power point presentation (such presentations can be found using a general search on the internet). Estimation – Pupils must learn to estimate – This way they will know when they have made an error. Subtract numbers to one and then two decimal places to begin with. Use the example of money to teach the concept e.g. £6.32 - <u>£4.11</u> Then extend so that decomposition is required. Give the children some completed questions to mark. All questions need to be written horizontally as well as in column form. Include incorrect answers.

5	**Calculate mentally a difference such as 8006 – 2993.**	Children will have been taught to use a number line and should be able to visualise this mentally. Some pupils may fail to recognise the steps they need to take and fail to add up ‘the steps’ at the end.	Why is it possible to solve this calculation mentally? How did you do it? If you counted backwards would it be possible to count up as well?	Work with number lines and ‘counting up’ to find a difference.
	Carry out column subtraction of positive integers less than 10,000.	Misconceptions occur when decomposing from a ‘high’ number. e.g. 9000 - 3654 Some pupils will attempt subtraction calculations using the formal written method, failing to recognise that it would be more efficient to calculate the answer mentally. Misconceptions occur when pupils (and teachers) use inaccurate language. e.g. 2367 - 1265 When talking about 2000 – 1000 they may refer to it as 2 – 1.	How can we ensure that we remember to answer the question? Should you answer this mentally or using a formal written method? Why? A range of questions to do with place value. What is the 3 digit worth? Is it more than the 2 in this number? What tips would you give someone to help them with column subtraction?	Give pupils a range of subtraction questions and ask them whether they would be better answered mentally or by a written method. Always refer to the digits accurately i.e. ‘take two hundred from three hundred’.

4	**Use known number facts and place value to subtract mentally, including any pair of two digit whole numbers.**		What strategies would you use to work out the answers to these calculations? Could you use a different method?	
	Carry out column subtraction of two integers less than 1000.	Pupils sometimes begin subtracting with the left hand column first. In tens and ones and other formal vertical subtraction calculations, children sometimes take the smaller unit number from the larger, regardless of whether it is part of the larger or smaller number. e.g. 945 - 237 712	Which column do we begin with? Why isn't it sensible to take the larger number from the smaller (5-7)? It is important that you don't say that it is impossible to take 5 from 7, as this is not true. What do we need to do instead? Where will we get our extra 10, 100 etc. from? What tips would you give someone to help them with column subtraction?	Practise using base ten materials and talk through the calculation. Teach composition, being careful to use the correct vocabulary. Demonstrate what is happening when we decompose, on an OHP with base ten materials. Show the tens and hundreds 'moving'. Teach composition using the expanded layout to begin with. This will help pupils who do not have a secure knowledge of place value. e.g. 64-28= 50 60 14 - 20 8 (this is often taught at the end of Yr 3)

3	**Know by heart all subtractions facts for each number to 20.** **Subtract mentally a 'near multiple of 10' to or from a two-digit number.**	If teachers use the phrase 'near multiple of ten' the children are often confused and believe that they should be multiplying a number. If they understand the term correctly then they might still struggle with compensating, not knowing whether to add or subtract. E.g. 46-19 = 46-20 +1 often confused as 46-20 – 1	What do we mean when we say 'a near multiple of 10?' Is this a good way of subtracting 17 or 18? If not why not? Why is this a good method? Show me on a 100 square.	Demonstrate the method on a number line. Use simpler terms to describe the operation e.g. 'take ten away and add one'. Encourage pupils to map out their calculations on their own number lines. This will help them to visualise what is happening and enable them to work more efficiently mentally.
2	**Understand that subtraction is the inverse of addition; state the subtraction corresponding to a given addition and vice versa.** **Know by heart all subtraction facts for each number to at least 10.**	Pupils not understanding the commutative law and believing that it is possible to change any addition or subtraction question around. e.g. 9+3=12 9-12=3	Can we change the calculation around and still get the same answer? I thought of a number. I subtracted 19 and the answer was 30. What was my number? How do you know?	Pupils need to know why they won't get the same answer if they change the calculation around. Demonstrate practically using materials such as counting blocks.

1	**Understand the operation of subtraction (as 'take away' or 'difference') and use the related vocabulary.** **Use mental strategies to solve simple problems using subtraction, explaining methods and reasoning orally.**	Pupils might not understand the concept of 'finding a difference'. This is largely due to the fact that they can count on or back and are unsure which method to choose. Not realising that numbers can be counted in order forwards and backwards.	Can we find the difference between two numbers by counting? Using a number line show me two numbers that have a difference of 2. How might you write that? Which number comes before / after 17? Does 16 always come before 17?	Consult your schools 'routes through' and concentrate on <u>either</u> counting forwards or backwards. Pupils often find it easier to 'count up' from a given number because many have consolidated their addition skills.
R	**Begin to relate subtraction to 'taking away'.** **In practical activities and discussion, begin to use the vocabulary involved in subtracting.**	That subtraction can only be described as 'taking away'	Explain to me what happens when we 'take away'. Make up a 'take away' question and show me how to do it.	Teach other phrases that mean the same as 'taking away'. How many less? Be careful that these introductions are made carefully as to much vocabulary at once will confuse pupils. Using a play house, start with 3 people in one room and four in another – 'which room has more people in it? How do you know? Move some people from one room to another – What has happened in this room?

1.3 Ordering Numbers

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Order a mixed set of numbers with up to three decimal places.**	Numbers with more digits are larger. e.g. 23.456 is larger than 123.5.	What did you look for first? Which part of each number did you look at to help you? What do you do when numbers have the same digit in the same place? Can you explain this to me using a number line?	**Remind pupils to always order numbers systematically, beginning with the left hand column.**
5	**Order a given set of positive and negative integers.**	Children find it difficult to understand zero because it represents, for them, something that does not exist. Numbers which represent quantities less than zero also represent the non-existent for many children and so are likely to pose problems for many of them. When ordering, the concept of 0 being greater than –1 is difficult for children to understand.	When ordering 201 and 210 why is it important to include the zeros? What does this number read (103)? What does the 0 tell us? Tell me two temperatures between 0°C and -10°C. Which is the warmer? How can you tell? How can something be less than zero? Can you think of any real life situations where this happens?	When the introduction is made, it should be through practical situations. This is likely to be the use of negative numbers indicating direction, either ‘down or back’. The understanding of ‘less than zero’ as a negative value can come later. Order numbers on a ‘washing line’. Demonstrate the concept using a thermometer and the example of a swimming pool, where the depths are shown in negative numbers.

4	**Use symbols correctly, including less than (<), greater than (>), equals (=).**	That > means ‘less than’ and < means ‘greater than’.	Look at this number sentence _+_=20. What could the missing numbers be? What is different about the number sentence _+_ = < 20? How would you choose numbers to make it correct?	Teach children that the largest number should always be placed next to the ‘largest’ end of the symbol. Some children find it helpful to think of the symbols as crocodile mouths.
3	**Read, write and order whole numbers to at least 1000; know what each digit represents.**	Confusion about the place value of numbers. Difficulties are especially apparent when ordering numbers such as 212 and 221. Failure to understand that the position of the numeral gives it the value.	When ordering numbers, where should we begin? The left hand column or the right hand column? When ordering a set of numbers what do you look for first? What does this number say (36)? What is the digit 3 worth? Is it worth more than the 6 in this number? If I change the digits around it reads 63. Is this number smaller or larger than 36?	Use place value ‘arrow’ cards to demonstrate to children how to partition numbers. Teach how to order numbers systematically beginning with the left hand column.
2	**Count, read, write and order whole numbers to at least 100; know what each digit represents (including 0 as a place holder).**	Reversal of digits is a common misconception. i.e. 03 for 30 or 31 for 13 etc. This creates problems when ordering numbers.	Show number cards 17 and 71. Which number says 17? How do you know? What does the other one say? How are they the same/different?	A range of activities designed to enable children to write all numbers to 100 correctly.

1	**Read, write and order numbers from 0 to at least 20; understand and use the vocabulary of comparing and ordering these numbers.**	Counting back and finding a number that is 'one less than'. Failure to understand that numbers can be expressed in different ways Pupils sometimes think they should add two numbers e.g. 12 = 1+2 = 3 therefore 4 is larger than 12. When counting objects, being confused by the mismatch between the number being said and the fact that only one object is being pointed to. Being confused by 'teen' numbers which are not read in the order that they are written, unlike the other numbers, for example 18 is read 'eighteen' not one-ty eight.	What number is one less than _? How could we write that number in words? Can you write the number 16? Listen carefully – 60 – Is this the same number? Does the digit '1' in '12' mean ten or one? Show cards with 12 and 21 on them. Which is 12? How do you know? If I take some objects away like this… will I have the same number left? What does this number say (17)? How could we write that number in words? Show cards with 15 and 51 on them. Which is 15? How do you know?	Practice counting backwards around the room. Challenge the pupils to do it faster each time Use place value cards to highlight the differences between tens and units.

R	**Use language such as more or less, greater or smaller, heavier or lighter, to compare two numbers or quantities.**	Confusion caused by vocabulary causes a great number of difficulties. Particular problems have been highlighted by the confusion between ‘more’ and ‘less’. Linking words with practical activities: It is common for reception pupils to think that the heavier object on a balance is the one that is ‘higher’ than the other.	**What would you rather have: £1 or £2? Why?**	Produce a vocabulary worksheet so that pupils can group words that mean the same thing. Put key words on the wall. In role play, give two children some sweets – Ask ‘is that fair? Why?’ Practical activities involving scales and balances. Pupils to label balances with the words ‘heavier’ and ‘lighter’. Pack two shopping bags with equal quantities but not equal weights. What is going to happen when we pick these up? Why?

1.4 Problem Solving

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Identify and use the appropriate operations (including combinations of operations) to solve word problems involving numbers and quantities, and explain methods and reasoning.**	An over-reliance on inadequate mental skills. Incorrect identification of the operation(s) to be used. Inadequate extraction of important information. Pupils miss the important information required to carry out the calculation(s).	What words do you look for in the wording of questions? What words mean you need to add/subtract/ multiply/divide?	Revise vocabulary. i.e. Ensure pupils understand what they are meant to do when the problem uses phrases such as 'how much more?' and 'What will be left?' Pupils underline / highlight the important information. Ask pupils to make up two different word problems for each of these calculations. Ask them to use a variety of words. (17+5) x 6 12.5 ÷ 5 – 0.25

5	**Use all four operations to solve simple word problems involving numbers and quantities, including time, explaining methods and reasoning.**	Incorrect identification of the operation(s) to be used. In a two-step operation, some pupils will be unsure of which operation to do first. Misconceptions arise because of confusion caused by different units of measurement.	How do you know whether you need to add/subtract Multiply or divide? Which operation will you need to do first? Why? What are the important things to remember when solving word problems?	
4	**Choose and use appropriate number operations and ways of calculating (mental, mental with jottings, pencil and paper) to solve problems.**	As problems become more complex pupils omit important steps.	What steps do we need to take to ensure that we get an accurate answer? How did you know that you need to add/subtract/ Multiply/divide?	Teach the 'seven steps to problem solving' (as outlined in the introduction).
3	**Choose and use appropriate operations (including multiplication and division) to solve word problems, explaining methods and reasoning.**	Not understanding which operation(s) is required to solve the problem. This is often due to a mis-understanding of vocabulary. e.g. Find total, more then, +, difference, less than, -, etc.	What are the important things to remember when solving word problems? How did you know that you need to add/subtract/ Multiply/divide?	Make certain that pupils know, understand and can recall the language, so that they can explain methods and reasoning.

2	**Choose and use appropriate operations and efficient calculation strategies to solve problems, explaining how the problem was solved.**		Give children a range of calculations. Which of these can you easily work out in your head? Which might you need to use jottings for?	Give children a means of jotting steps.
1	**Use mental strategies to solve simple problems using counting, addition, subtraction, doubling and halving, explaining methods and reasoning orally.**	Not knowing what the problem is asking them to do.	What do you need to find out? How do you know that you need to add/subtract/double? What clues are there? What did you do in your head first? How did you work it out?	
R	**Use developing mathematical ideas and methods to solve practical problems.**	That most problems cannot be solved.	What is the problem? What shall we do?	With six children say 'there aren't enough chairs around the table for all of us. What shall we do?' Some children might suggest getting extra chairs or sitting on the carpet. They will realise that problems can be solved in a variety of ways.

1.5 Multiplication

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Multiply decimals mentally by 10 or 100, and explain the effect**	Misunderstand the concept of making a number 10/100/1000 times bigger, prefer to learn 'add a zero'. Causes difficulties when working with decimal numbers and fractions.	*"The calculator display shows 0.1. Tell me what will happen when I multiply by 100. What will the display show?" "What number is 10 times as big as 0.01? How do you know?" "How would you explain to someone how to multiply by 10?"*	Label chairs TH, H, T, O and choose children to sit, holding a digit card. When multiplying by 10/100/1000, the children move required no. spaces along chairs. Additional children will be needed holding 'zero' cards as spare chairs become available from the units. Question pupils as to their value, what value do they have now they have moved seats? How many times larger are you?
	Carry out short multiplication of numbers involving decimals	Ignore decimal point, perform calculation, then 'count how many digits after the point'. Effective shortcut, but difficulty when applying to mental work – encourage 'why does it work?'	*"The answer is 15.2. Make up some questions using multiplication with decimal numbers that could give this answer."*	Encourage the children to approximate first, e.g. 4.92x3.1 is approximately 5x3, so answer should be approximately 15. Start with mental strategies first…25 x 0.4 is 10 times smaller than 25 x 4, i.e. 10 times smaller than 100, = 10. *Use the 'grid method' as it is based upon partitioning, with which the pupils will be extremely familiar. It is worth showing the pupils practically, with cubes, that multiplying the parts is the same as multiplying by the whole number in one step.*
	Carry out long multiplication of a three-digit by a two-digit number	Children introduced to formal written strategy too early, when 'stuck' reach for a calculator because have no strategy of their own.	Give the children three or four long multiplications with mistakes. Ask them to identify the mistakes and talk through what is wrong and how they should be corrected.	Use 20 x 5 as a key fact and then extend to 20 x 50 which is 10x bigger. Say the number sentences one after the other; *Twenty times five is one hundred* *Twenty times fifty is one thousand* Write the connected number sentences one above the other,

		Place value errors when performing written calculations can cause problems even for able pupils. Children are taught to multiply single digits and count the number of zeros. 20 x 50 = 100 is a common mistake as children don't know what to do with the 'extra' zero	Give a multiplication question (147x32) calculated by both the grid method and long multiplication. Ask questions like *"what 2 numbers multiplied together give 4410? Or 294?"*	20 x 5 = 100 20 x 50 = 1000

5	**Multiply any positive integer up to 10000 by 10 or 100 and understand the effect**	Pupils do not understand that x10 and then x 10 again, is the same as x100. Prefer to learn ‘add a zero’ and so limited understanding.	*“Why do 6x100 and 60x10 give the same answer?”* *“I have 37 on my calculator display. What single multiplication should I key in to change it to 3700? Why does it work?”*	See Year 6 activity involving children moving places along a set of labelled chairs.
	Know by heart all multiplication facts up to 10x10	Children not understand the meaning of ‘lots of’ or ‘groups of’. Children see it as a test of their memory, not linking tables facts.	*“If someone had forgotten their 8 times table, what tips could you give them to work it out?”* *“What other links between times tables are useful?”*	Help the children see the links between the tables. Use a multiplication grid and complete the easier questions. Encourage them to learn the square numbers (4x4, 5x5 etc…) They will be shocked to see that all of the tables can be reduced to just a few facts to learn! Play games requiring tables knowledge, use software such as ‘Developing Number’, which encourages the use of strategies.
	Carry out short multiplication of a three-digit by a single-digit integer **Carry out long multiplication of a two-digit by a two-digit integer**	Children are introduced to formal written methods before fully understand the concept, becomes a test of their memory to remember the ‘rule’, and have no strategies to rely upon when they are ‘stuck’. Problems with place value can cause difficulties with written work.	*“Roughly what answer do you expect to get? How did you reach that estimate?”* *“Do you expect your answer to be less than or greater than your estimate? Why?”* Give the children some worked examples that are incorrect. *“Is this correct? How do you know? How could we put it right?”*	When introducing multiplication with larger numbers, revert back to an earlier, more secure written method, to increase confidence. Only move to a more formal method when secure. Try partitioning the numbers and dealing with them in parts – the grid method supports this, and many children may never advance their written method beyond this. Encourage mental/jotting approximation before starting written work.

		Children need to understand the connection between 6 x 3 and 60 x 3, understanding that the answer is 10x bigger because the number being multiplied is 10x bigger.	Give question such as 37x14 calculated by both the grid and the long multiplication method. Ask questions like “*what two numbers multiplied together have the answer 370?”* *“How would it be different if I worked out 14x37?”*	Use a counting stick to count in multiples of a number and then the corresponding multiple of 10, e.g. 3, 6, 9, 12….30, 60, 90, 120…. Use multiplication grids with multiples of 10 on, e.g. instead of 3 x 4, 30 x 4 etc.

4	**Know by heart facts for the 2, 3, 4, 5, and 10 multiplication tables**	See Year 5	See Year 5 *“The product is 40, what two numbers could have been multiplied together?”*	See Year 5 Play ‘Round the World’; One child stands behind the chair of another, the only 2 that can answer a question given. Should the standing child answer first, then move to next chair – aiming to get around the whole class (the world). Should the seated child answer first, then swap places and continue.
3	**Know by heart facts for the 2, 5 and 10 multiplication tables**	See Year 4/5	See Year 4/5 Show some missing number statements such as □ x 5 = 35 and 10 x □ = 90… “*What’s the missing number – how do you know?”* Show □ x O = 30 and ask “*what could the missing numbers be?”*	See Year 4/5 Children may need to go back to multiplication as an array, or repeated addition, to gain security with the notion of multiplication.

2	**Understand the operation of multiplication as repeated addition or as describing an array**	Children inaccurate when displaying arrays of cubes/objects and so pattern is not clear. Link not clear between the array and the seemingly abstract number given as the answer.	*"How many dots are there? How can you work it out without counting them all?" What number sentence can you write to record your method using x…+…?" "Here are 12 (or 20, 24) counters. Can you arrange them in equal rows? Record in a number sentence."*	Useful to arrange objects on a grid base, so that pupils put one object in each square, avoids confusion over rows and columns. These could be numbered at the sides. Lots of practical examples required, turn grid round, so understand multiplication can be done in any order.
	Know by heart facts for the 2 and 10 multiplication tables	See Year 3/4/5 Confusion occurs because spoken numbers e.g. sixty, seventy, eighty etc. follow a regular pattern which link to the single digit numbers. Ten, twenty, thirty, do not relate directly to their corresponding single digit number	See Year 3/4/5	See year 3/4/5 Continue the counting even if pupils get the first few numbers confused as the later numbers are easier. Encourage children to chant as a class/group, point to the numbers on the 100 square.

1.6 Division

Year	Objective	Misconception	Key Questions	Teaching Activity
6	**Derive quickly division facts corresponding to multiplication tables up to 10 x 10**	Lack of understanding that division is grouping as well as sharing. Lack of tables knowledge.	Start with a number with at least 6 factors, e.g. 56. "How many different X and ÷ facts can you make using what you know about 56?" "What if you started with 5.6?"	Lots of activities requiring constant repetition of tables, play 'Round the World'; One child stands behind the chair of another, the only 2 that can answer a question given. Should the standing child answer first, then move to next chair – aiming to get around the whole class (the world). Should the seated child answer first, then swap places and continue.
	Carry out short division of numbers involving decimals	Ignore decimal point when calculating, then simply 'slot back in'. Comes from over generalisation of adding decimals.	"The answer is 12.6. What questions could you ask using division with decimal numbers?"	
	Divide decimals mentally by 10 or 100, and integers by 1000, and explain the effect	(included above)	"Why do 5 ÷ 10 and 50 ÷ 100 give the same answer?"	See Year 5. When operating with decimal numbers, and whole numbers where ones digit is not zero, choose another child to sit and hold the 'decimal point' card. They will NEVER move! Additional chairs will be required for the tenths, hundredths columns.

		Misunderstand the concept of making a no. 10/100/1000 times smaller, prefer to learn ‘knock off a zero’. When the number ends in a different digit, simply knock that off. Ignore decimal point, or ‘move it’ - often taught by parents!	“I divide a number by 10, and then again by 10. The answer is 0.3. What number did I start with? How do you know?” “The calculator display shows 0.1. Tell me what will happen when I multiply by 100? What will the display show?” “How would you explain to someone how to multiply a decimal by 10?”	

5	**Divide any positive integer up to 10000 by 10 or 100 and understand the effect**	Pupils do not understand that ÷ 10 and then ÷ 10 again, is the same as ÷ 100.	“Why do 30 ÷ 10 and 300 ÷ 100 give the same answer?” “I have 3700 in my calculator display. What single division should I key in to change it to 37? Explain why this works.”	Label chairs TH, H, T, O and choose children to sit, holding a digit card. When dividing by 10/100/1000, the children move required number of spaces along chairs. Child as zero ones ‘drops off’ end. NOTE; only used when ones digit is zero. Question pupils as to their value, what value do they have now they have moved seats? How many times smaller are you?
	Carry out short division of a three digit by a single digit integer	See Year 6 Pupils are introduced to written method before fully understanding the concept of grouping or ‘chunking’. Need more concrete examples. When dealing with remainders, pupils have little understanding of how to represent as a fraction or a decimal.	“Roughly what answer do you expect to get? How did you come to that estimate?” “Do you expect your answer to be less than or greater than your estimate – why?”	Ensure that the pupils relate the division to multiplication; 27÷3 … ‘how many chunks of 3 are there in 27?’ Count up in 3s. Less able children use a tables square for multiplication facts, so not to slow down understanding of the division process.

4	**Quickly derive division facts corresponding to the 2, 3, 4, 5, 10 multiplication tables**	Lack of understanding that division is grouping as well as sharing. Lack of tables knowledge. Not understand the concept of ‘inverse’.	How many division facts can you make using what you know about 24 (or 20, 30…). How did you work out the division facts?	Lots of practise repeating multiplication tables, play games such as ‘round the world’ (see Year 6). Show the pupils, physically, that ‘groups of’ (multiplication) and division are the same, using cubes. Use multiplication grid for ‘facts’, so not slowing down division process.
	Find remainders after division	Lack of understanding of WHAT the remainder actually represents, need to see calculation in concrete way, i.e. cubes left over after division. Ignoring the context of the question – should it be rounded up or down?	“Do all divisions have remainders?” “Make up some division questions that have a remainder of 1" ”How did you do it?” “Make up some division questions that have no remainder. How did you do this? Why do they not have a remainder?”	Practical examples where children need to put objects in boxes (egg boxes good!), alongside written form of the division question. Lots of practical examples of remainders – e.g. coaches for swimming etc.

3	**Understand division and recognise that division is the inverse of multiplication**	Children not see link between the two operations, need more experience of chunking, as well as sharing. Need to be taught alongside each other – not separately. Children think that 2 ÷ 4 is the same as 4 ÷ 2.	"What is the answer to 20 ÷ 5? Can you make up a problem that means you need to work out 20 ÷ 5 to solve it?" "Can you tell me some numbers that will ÷ exactly by 2? 5? 10? How do you know?" Give the children a set of numbers that are related by x and ÷ facts along with the multiplication, division and equals signs. Ask them to form some x and ÷ statements. Ask them to match the ones that are linked in some way and to explain why. "If I multiply a no. by 2, and then ÷ the answer by 2, what happens?" "Is 2 ÷ 4 the same as 4 ÷ 2?" Why not?	Show children physically that 2÷4 cannot be the same as 4÷2… use a 2m length of wool, cut into 4 equal pieces. Use a 4m length of wool, cut into 2 pieces – are they the same? Lots of examples using cubes to show that multiplication and division are the inverse of each other. When teaching the chunking written method, model with cubes every step of the written process.
2	❖ **Know and use halving as the inverse of doubling**	Children confuse the words 'halving' and 'doubling'. Lack of understanding that two operations are linked, often taught separately.	"I'm thinking of a number, I've halved it and the answer is 15. What number was I thinking of? Explain how you know?" "I'm thinking of a number. I've doubled it and the answer is 18. What number was I thinking of? How do you know?"	Practise using the vocabulary with the pupils – link 'doubling' to a 'double decker bus', they will remember which means twice as many! Show practical examples of halving an apple, and doubling the number of apples etc.